

chapter 18

Electric Charge

Three basic observations:

- 1) material is usually neutral
 - 2) charge is conserved
 - 3) charge is quantized.
-

1. Neutrality: most naturally occurring material is electrically neutral

which means one of two things:

either \rightarrow charges do not exist

or neutrality means no net exists.

A. this leads to the idea that point charges exist
by experiment we find two types of charge:
positive/negative

OR positive/not positive

OR right handed/left handed

all lead to the ability to say $\sum q_i = 0$.

B. observed interactions:

like charges mutually repel each other with a force

and unlike charges mutually attract each other with a force

"mutually" = Newton's 3rd pairs

2. Conservation of charge: $Q_{tot\ i} = Q_{tot\ f}$

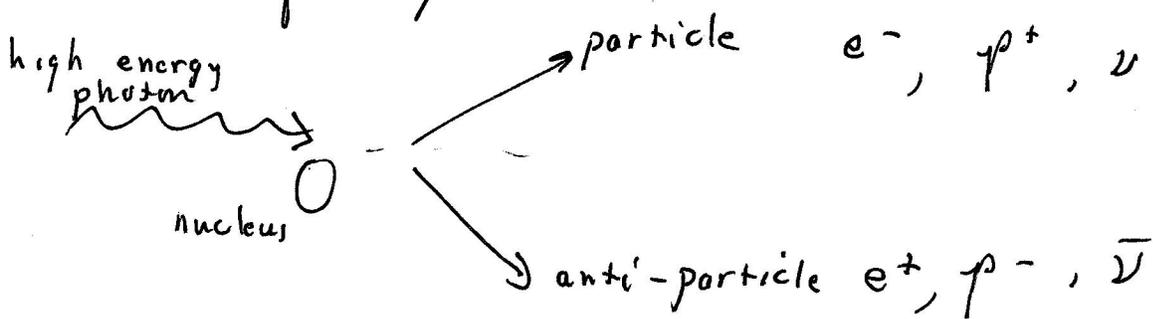
A. \hookrightarrow this leads to "charge separation"

rubber rod + fur

glass rod + synthetic cloth

$i =$ neutral
 $f =$ rub + find
 oppositely charged
 \hookrightarrow put together \rightarrow back to neutral

B. creation of charge:



annihilation



3. Quantized charge: $e =$ fundamental charge
 $= 1.60 \times 10^{-19}$ Coulombs

$$q_{e^-} = -e$$

$$q_{p^+} = +e$$

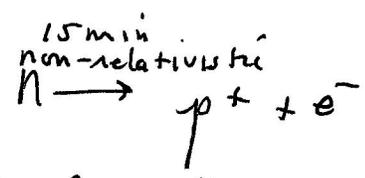
$Q_{net} =$ excess numbers of electrons
 or protons

$$= N(\pm e)$$

\hookrightarrow integer

	charge	mass
electron	$-e$ $-1.60 \times 10^{-19} \text{ C}$	$9.11 \times 10^{-31} \text{ kg}$
proton	$+e$ $+1.60 \times 10^{-19}$	$1.67 \times 10^{-27} \text{ kg}$
neutron	0	$1.67 \times 10^{-27} \text{ kg}$

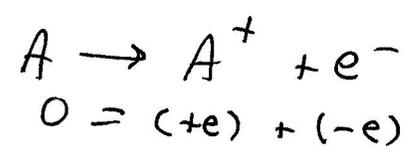
$$m_e = \frac{1}{1836} m_p.$$



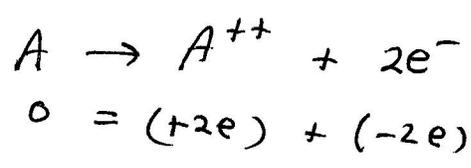
$$\therefore m_n \sim m_p$$

ionization states:

singly ionized = 1 electron removed



doubly ionized \rightarrow 2 electrons removed



terms

1. Conductors = "electrics" = easily permits the flow of charges
2. non-conductors = "dielectrics" = do not easily permit the flow of charge
= "insulators"
3. semi-conductor: under the right conditions or modifications, the material can permit the flow of charges: i.e. p-n junction diode

actually what drives these definitions is quantum mechanics + the Pauli Exclusion Principle: no two electrons can have the same set of quantum numbers

conductors = electrons have lots of accessible quantum states

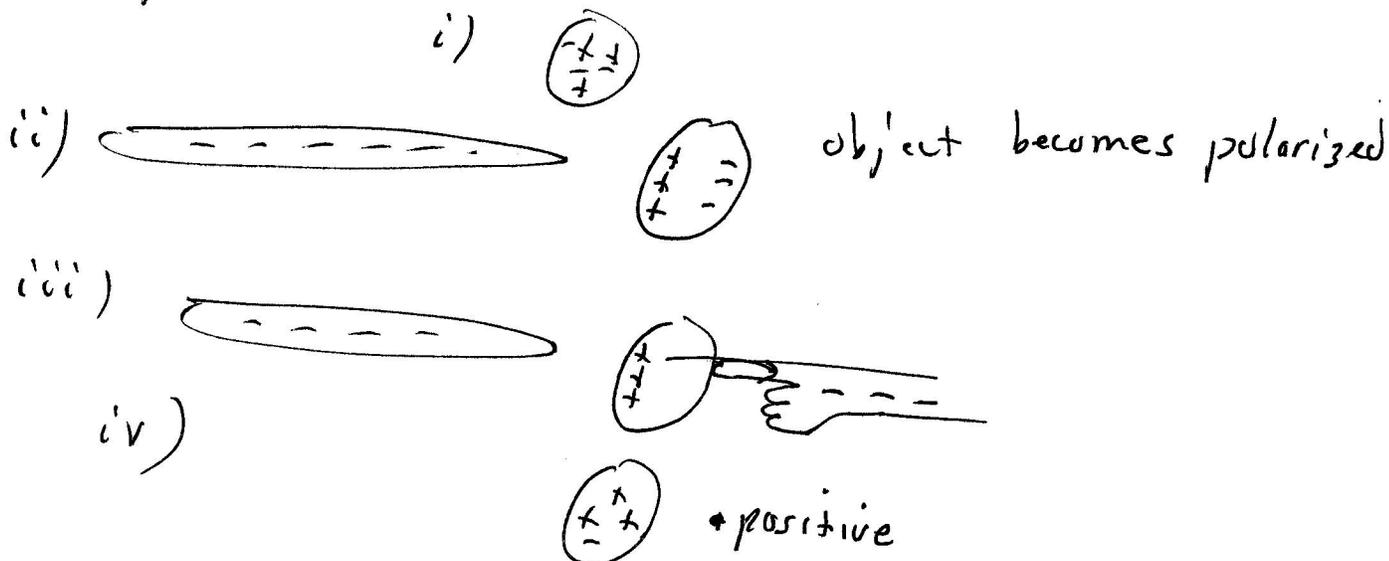
non-conductors = have few or no accessible quantum states

semi-conductors = by adding impurities, extra states are made available

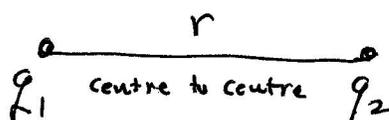
1) Direct charging / charge separation:



2) charging by induction



Coulomb's Law (1736 - 1806)



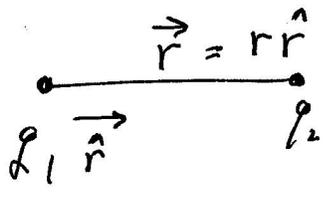
$$F \propto \text{charge} \\ \propto \frac{1}{r^2}$$

$$F_e \propto \frac{q_1 q_2}{r^2}$$

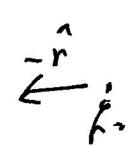
$$F_e = k \frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

$$\epsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$$

Specifically: $\vec{F}_{on\ q_2}$ due to q_1 

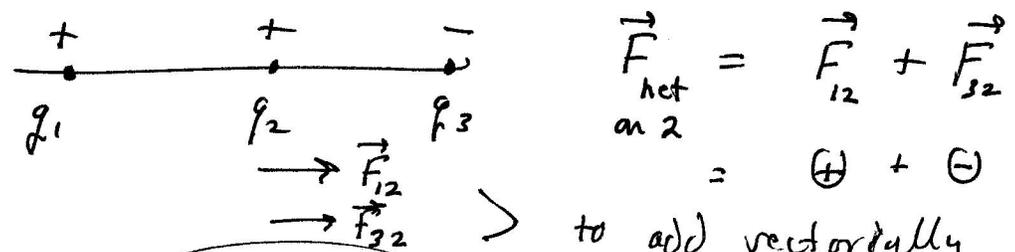
$$\vec{F}_{on\ 2} = k \frac{q_1 q_2}{r^2} \hat{r}$$

	q_1	q_2	
	+	-	$-\hat{r}$
	-	+	$-\hat{r}$ attract
	+	+	$+\hat{r}$ repel
	-	-	$+\hat{r}$ repel

* so the "sign" of the force tells if the force is attractive or repulsive

* there is no sense to a negative magnitude!

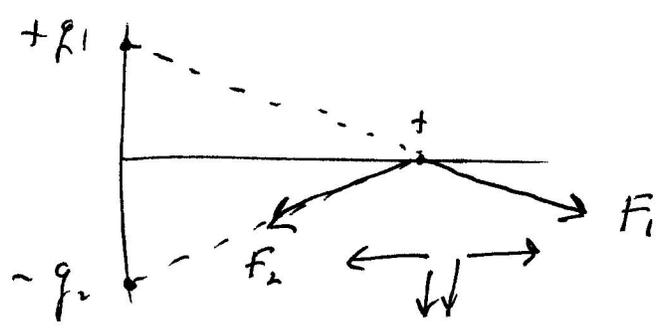
i.e.



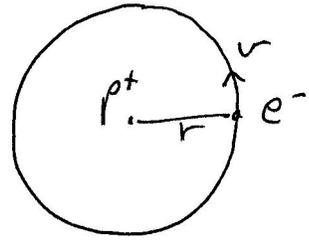
let \rightarrow x

> to add vectorially sign convention

$$F_{net} = |\vec{F}_{12}| + |\vec{F}_{32}|$$



Hydrogen atom (Bohr Model): electron orbits proton with radius $r = 5.29 \times 10^{-11} \text{ m}$
 find the speed of the electron:



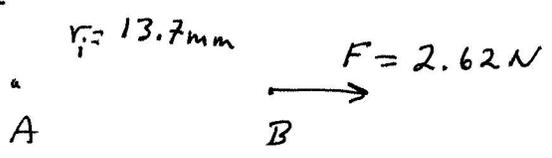
electrostatic (Coulomb) force provides the required centripetal force

$$F_e = F_c$$

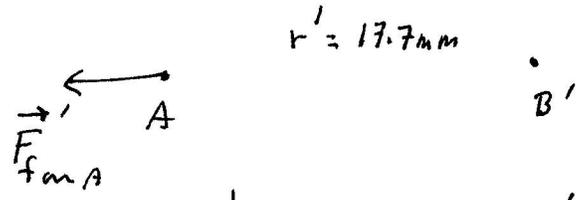
$$\frac{ke^2}{r^2} = \frac{mv^2}{r} = m\omega^2 r = m(2\pi f)^2 r$$

$$v^2 = \frac{ke^2}{rm} \rightarrow v = \sqrt{\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(5.29 \times 10^{-11})(9.11 \times 10^{-31})}} = 2.19 \times 10^6 \text{ m/s}$$

15.2



$$k \frac{q_A q_B}{r_i^2} = 2.62 \text{ N}$$

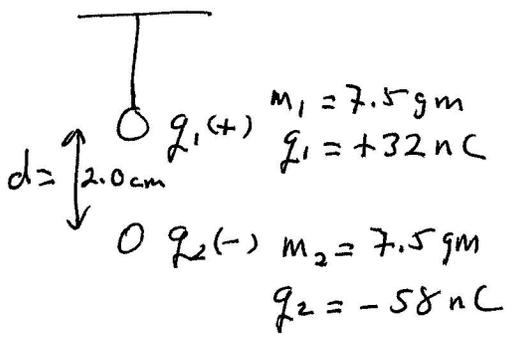


finally: $\frac{k q_A q_B}{r'^2} = F'_f \rightarrow \vec{F}'_A = -\vec{F}'_B$
N3LP

$$k \frac{q_A q_B}{r_i^2} = 2.62 \text{ N} \left(\frac{13.7 \text{ mm}}{17.7 \text{ mm}} \right)^2 = F'_f \left(\frac{17.7 \text{ mm}}{17.7 \text{ mm}} \right)^2$$

$$F'_f = (2.62 \text{ N}) \left(\frac{13.7 \text{ mm}}{17.7 \text{ mm}} \right)^2 = 1.57 \text{ N to the left on A}$$

15.4



in eqm: $F_{\text{net}} = 0$, let $\uparrow +$

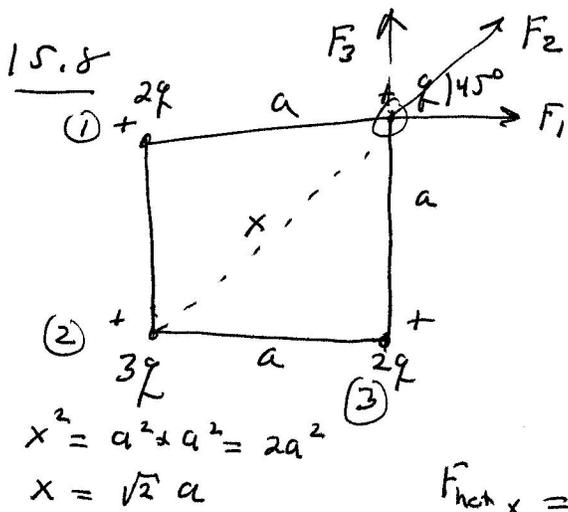
$$T - m_1 g - F_e = 0$$

$$T = m_1 g + F_e$$

$$T = (7.5 \times 10^{-3} \text{ kg})(9.8) + \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(32 \times 10^{-9} \text{ C})(58 \times 10^{-9} \text{ C})}{(0.02 \text{ m})^2}$$

$$T = 7.35 \times 10^{-2} \text{ N} + 4.18 \times 10^{-2} \text{ N}$$

$$T = .115 \text{ N}$$



15.7

$$|F_1| = |F_3| = \frac{k(2q)q}{a^2} = \frac{2kq^2}{a^2}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3, \text{ let } \uparrow +$$

$$F_{\text{net}_x} = F_1 + F_2 \cos 45^\circ$$

$$F_{\text{net}_y} = F_3 + F_2 \sin 45^\circ$$

$$F_2 = \frac{k(3q)q}{x^2} = \frac{3kq^2}{2a^2}$$

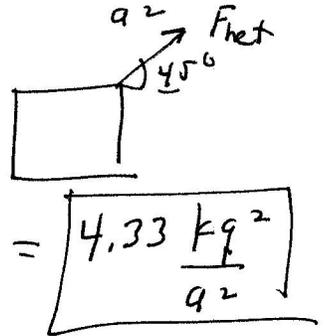
$$F_{\text{net}_x} = \frac{2kq^2}{a^2} + \frac{3kq^2}{2a^2} \cdot \cos 45^\circ = \left(2 + \frac{3}{2}(0.707)\right) \frac{kq^2}{a^2}$$

$$= 3.06 \frac{kq^2}{a^2}$$

$$F_{\text{net}_y} = \frac{2kq^2}{a^2} + \frac{3kq^2}{2a^2} \cdot \sin 45^\circ = 3.06 \frac{kq^2}{a^2}$$

$F_{\text{net}_x} = F_{\text{net}_y} \Rightarrow$ sum lies along diagonal

$$F_{\text{net}} = \sqrt{F_{\text{net}_x}^2 + F_{\text{net}_y}^2} = \sqrt{(3.06)^2 + (3.06)^2} \frac{kq^2}{a^2}$$

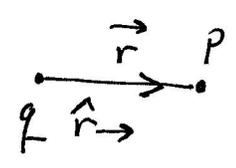


Electric Field

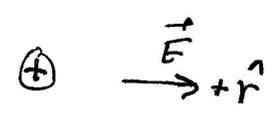
The electric field is a vector field that propagates at the speed of light from charges, influencing the space around them.

defined: $\vec{E} = \frac{\vec{F}_e}{q_0} \longrightarrow$ or a charge in an electric field experiences a force $\vec{F}_e = q \vec{E} \longrightarrow m\vec{a}$

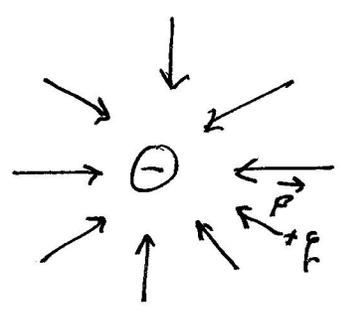
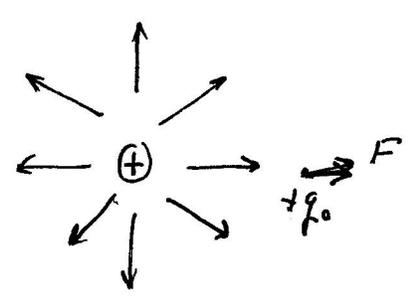
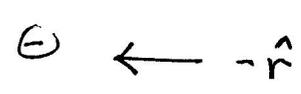
for a point charge: $\vec{E} = \frac{k q q_0}{r^2} \hat{r} = \frac{k q}{r^2} \hat{r}$



for +q, \vec{E} points parallel to \hat{r} (away)

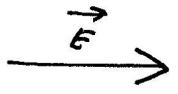


for -q, \vec{E} points radially toward the q (opposite to \hat{r})



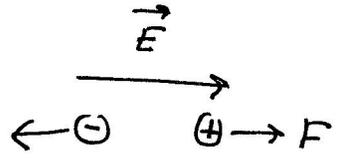
Michael Faraday imagined these electric field lines to describe the influence of charges on the space around them.

Qualitatively, electric field lines are shown as vectors



such that

1.



positive charges feel a force in the direction of \vec{E} and negative charges feel a force opposite to \vec{E}

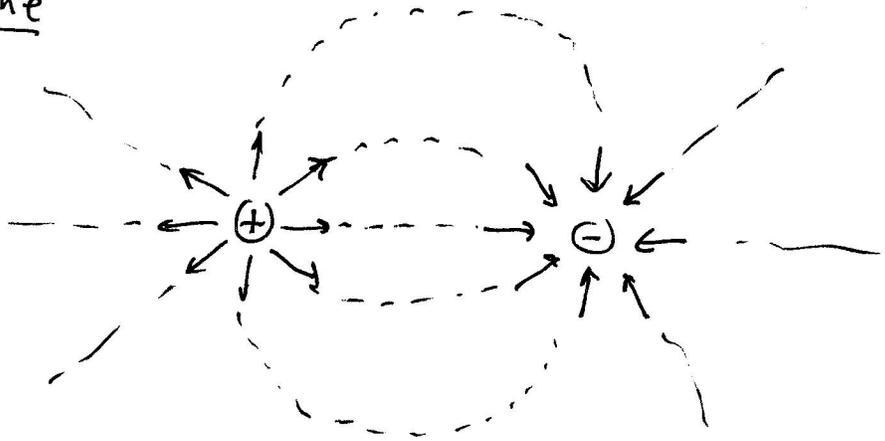
2. the density of lines (# lines/m²)

\propto electric field strength

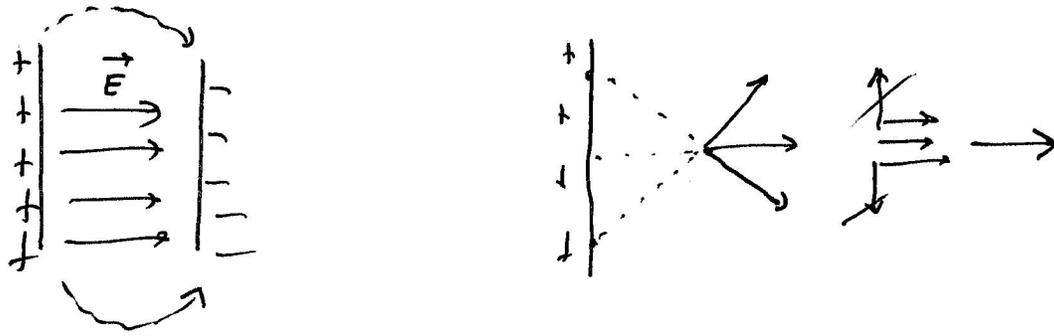
3. the total number of lines

\propto net charge inside the surface enclosing

Imagine



oppositely charged, parallel plates

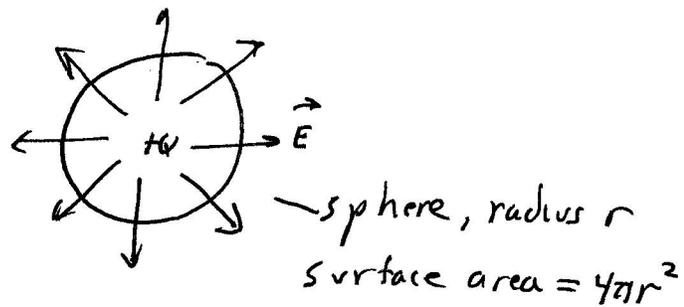


$$\sigma = \text{surface charge density} \\ = \frac{Q}{A} \text{ C/m}^2$$

$$\Rightarrow E \text{ (between the plates)} = \frac{\sigma}{\epsilon_0}$$

Gauss' Law

for a point charge, $+Q$
the electric field is radial
and has an equal value at
any point at the surface of a
sphere, centered on Q , of radius r



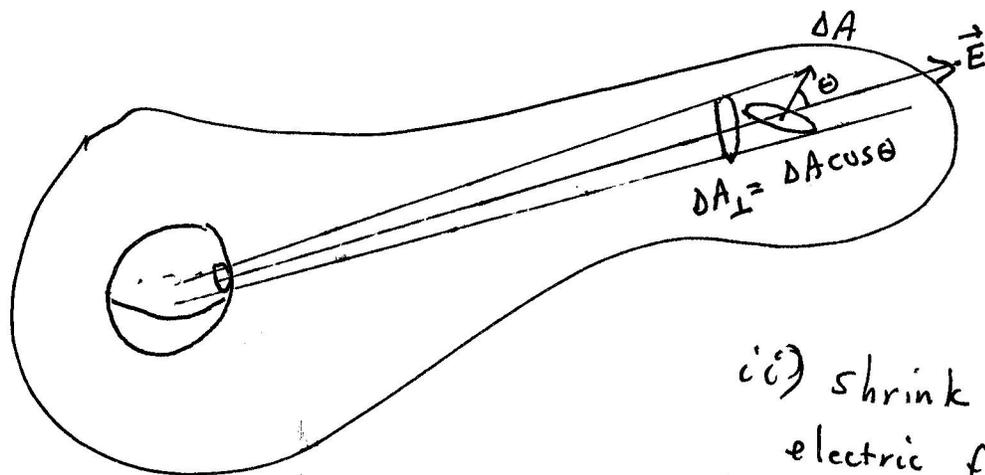
$$E = \frac{kQ}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \text{ N/C} \rightarrow \text{draw to scale, let } 1 \text{ N/C} = 1 \text{ line/m}^2$$

$$\# \text{ lines} = \# \text{ lines/Area} \times \text{surface Area}$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r^2} \right) \frac{\text{lines}}{\text{m}^2} \times 4\pi r^2 (\text{m}^2) = \frac{Q}{\epsilon_0}$$

$$\boxed{\text{total \# of lines} = \frac{Q}{\epsilon_0}}$$

↓
total electric flux



i) rotate area element ΔA so it is \perp to E

$$\rightarrow \Delta A_{\perp} = \Delta A \cos \theta$$

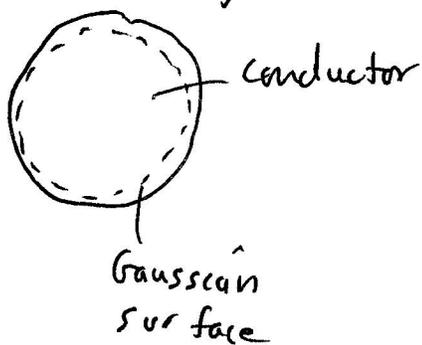
ii) shrink ΔA_{\perp} down the electric field line and map on surface of sphere

iii) we know for sphere that total flux = $\frac{Q}{\epsilon_0}$

$$\Delta \phi = E \cdot \Delta A_{\perp} = E \Delta A \cos \theta$$

$$\phi_{\text{total}} = \sum E \Delta A \cos \theta = \frac{Q_{\text{net}}}{\epsilon_0} = \text{Gauss' Law} = \vec{E} \cdot \vec{\Delta A}$$

Interesting problem:



1. add excess charge to neutral conductor
2. Let the charge distribute it self until eqm. is achieved

$$\Rightarrow F_e \text{ on any } Q = \phi = Q E$$

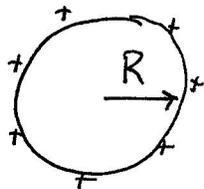
$$\therefore Q E = 0 \rightarrow \therefore E = 0$$

3. Choose Gaussian surface just under the surface of the conductor

$$4. \text{ Since } \sum \vec{E} \cdot \vec{A} = \frac{Q_{\text{inside}}}{\epsilon_0}$$

5. if $\vec{E} = 0$ then $Q = 0 \Rightarrow$ which means the excess charge is not inside $\rightarrow \therefore$ it must be on the surface!

charged hollow spherical shell



i) for $r > R$ (outside the shell) let gaussian surface be a sphere

$$\sum \vec{E} \cdot \vec{A} = E (4\pi r^2) = \frac{Q_{in}}{\epsilon_0} \rightarrow \left(E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2} \right)$$

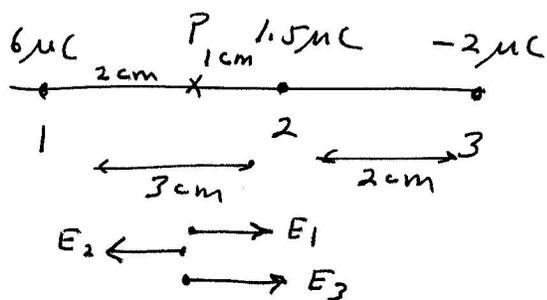
assumes all the charge is at the centre of the sphere

ii) for $r < R$ (inside the sphere)

$$\sum \vec{E} \cdot \vec{A} = E (4\pi r^2) = \frac{Q_{in}}{\epsilon_0}, \text{ but } Q_{inside} = 0 \text{ (shell is hollow)}$$

$$= 0 \therefore \boxed{E = 0 \text{ inside}}$$

15.18



$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

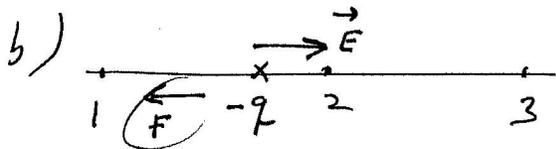
$$\text{let } (\rightarrow +) \quad E_p = E_1 - E_2 + E_3$$

$$E_p = \frac{kq_1}{(0.02\text{m})^2} - \frac{kq_2}{(0.01\text{m})^2} + \frac{kq_3}{(0.03\text{m})^2}$$

$$E_p = \frac{(9 \times 10^9)(6 \times 10^{-6})}{(0.02\text{m})^2} - \frac{(9 \times 10^9)(1.5 \times 10^{-6})}{(0.01\text{m})^2} + \frac{(9 \times 10^9)(2 \times 10^{-6})}{(0.03\text{m})^2}$$

$$E_p = 1.35 \times 10^8 \text{ N/C} - 1.35 \times 10^8 \text{ N/C} + 2.00 \times 10^7 \text{ N/C}$$

$$= \boxed{2.0 \times 10^7 \text{ N/C} \rightarrow} \text{ since answer is } \oplus$$



$$\vec{F} = -q \vec{E} = (-2.0 \times 10^{-6} \text{ C})(2.0 \times 10^7 \text{ N/C}) = \boxed{-40 \text{ N}}$$

↳ to the left

↳ opposite direction to E. field

15.20

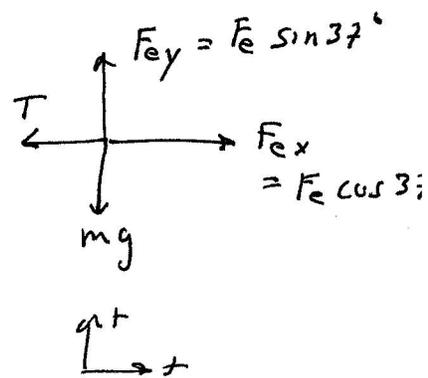
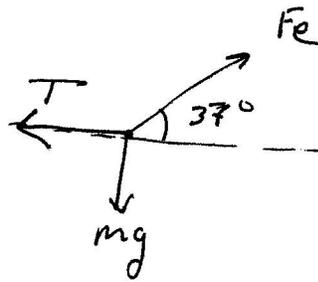
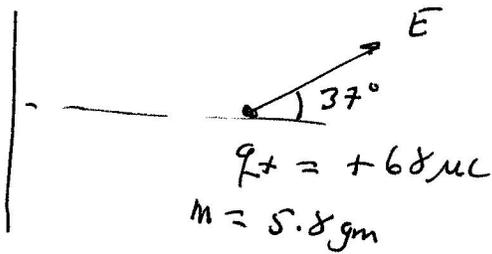
$$a) \quad \xrightarrow{E = 300 \text{ N/C}}$$

$$\xleftarrow{F_e} e^- \quad F = eE = ma \rightarrow a = \frac{eE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(300 \frac{\text{N}}{\text{C}})}{9.11 \times 10^{-31} \text{ kg}}$$

$$a = 5.27 \times 10^{13} \text{ m/s}^2$$

$$b) \quad v_f = v_0 + at = (5.27 \times 10^{13} \text{ m/s}^2)(1.0 \times 10^{-8} \text{ s}) = 5.27 \times 10^5 \text{ m/s}$$

15.22



$$F_{ey} - mg = 0 \rightarrow F_e \sin 37^\circ = mg$$

$$F_{ex} - T = 0 \rightarrow F_e \cos 37^\circ = T$$

$$\therefore \tan 37^\circ = \frac{mg}{T} \rightarrow T = \frac{mg}{\tan 37^\circ}$$

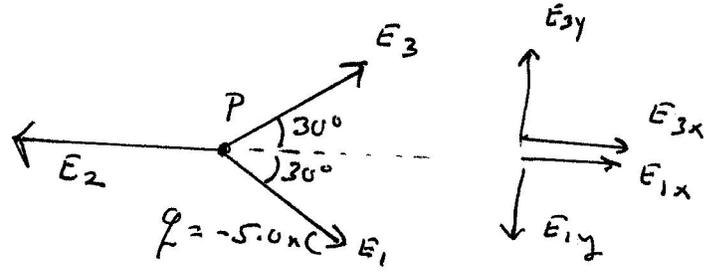
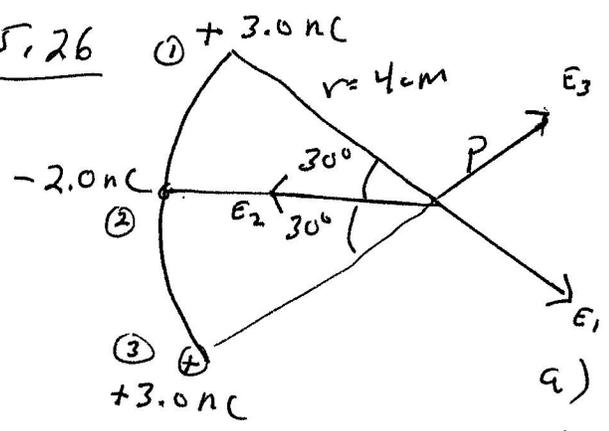
$$T = \frac{(5.8 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)}{\tan 37^\circ} = 7.54 \times 10^{-2} \text{ N}$$

from x-component

$$F_e = qE = \frac{T}{\cos 37^\circ}$$

$$E = \frac{T}{q \cos 37^\circ} = \frac{7.54 \times 10^{-2} \text{ N}}{(68 \times 10^{-6} \text{ C}) \cos 37^\circ} = 1.39 \times 10^3 \text{ N/C}$$

15.26



a) since $|E_1| = |E_3| \rightarrow$ same y-components
 $E_{1y} + E_{2y} = 0$
 but $\rightarrow +$

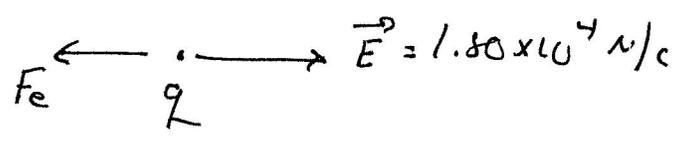
$$E_{net,x} = E_{1x} + E_{3x} - E_2, \quad E_{1x} + E_{2x} = 2E_1 \cos 30^\circ$$

$$E_p = 2 \left[\frac{(9 \times 10^9)(3 \times 10^{-9} \text{ C})}{(0.04 \text{ m})^2} \cdot \cos 30^\circ \right] - \frac{(9 \times 10^9)(2 \times 10^{-9} \text{ C})}{(0.04 \text{ m})^2}$$

$$= 2.92 \times 10^4 \text{ N/C} - 1.13 \times 10^4 \text{ N/C} = \boxed{+1.80 \times 10^4 \text{ N/C}}$$

↳ to right

b) F_{net} on $q = -5.0 \text{ nC}$ at P.

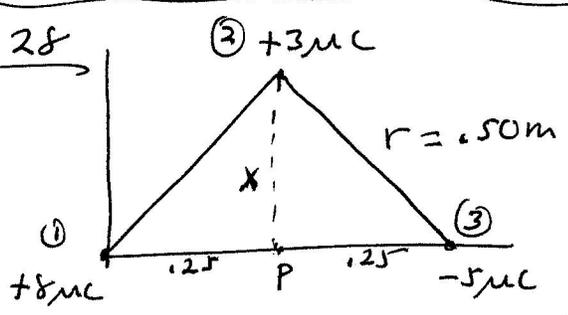


$$F_e = (-5.0 \times 10^{-9} \text{ C})(1.80 \times 10^4 \text{ N/C})$$

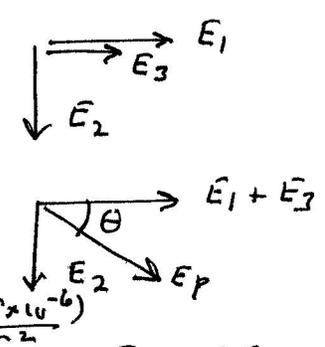
$$F_e = -9.0 \times 10^{-5} \text{ N}$$

to the left

15.28



at P



$$x = 0.50 \text{ m} \sin 60^\circ = 0.433 \text{ m}$$

$$E_x = E_1 + E_3 = \frac{(9 \times 10^9)(8 \times 10^{-6})}{(0.25)^2} + \frac{(9 \times 10^9)(5 \times 10^{-6})}{(0.25)^2}$$

$$= 1.15 \times 10^6 \text{ N/C} + 7.20 \times 10^5 = 1.87 \times 10^6 \text{ N/C}$$

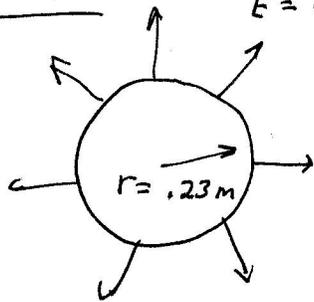
$$E_y = E_2 = \frac{(9 \times 10^9)(3 \times 10^{-6})}{(0.433)^2} = 1.44 \times 10^5 \text{ N/C}$$

$$E_p = \sqrt{E_x^2 + E_y^2} = \sqrt{(1.87 \times 10^6)^2 + (1.44 \times 10^5)^2} = \boxed{1.88 \times 10^6 \text{ N/C}}$$

$$\tan \theta = \frac{E_y}{E_x + E_3} = \frac{1.44 \times 10^5}{1.87 \times 10^6} = 0.077 \rightarrow \theta = 4.4^\circ$$

below x-axis

15.42



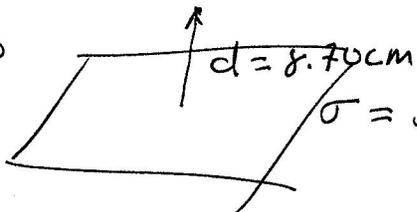
$E = 575 \text{ N/C}$

$E = \frac{kQ}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{(575 \frac{\text{N}}{\text{C}})(.23)^2}{9 \times 10^9}$

$Q = +3.38 \times 10^{-9} \text{ C} = +3.38 \text{ nC}$

$Q_{\text{net inside}} = +3.38 \text{ nC}$, distributed somehow - we cannot tell.

15.50



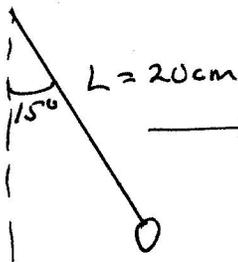
$\sigma = 5.20 \times 10^{-6} \text{ C/m}^2$

if the sheet is large enough or if "d" is small compared to the sheet

then $E = \frac{\sigma}{2\epsilon_0}$ and is uniform

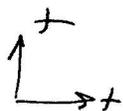
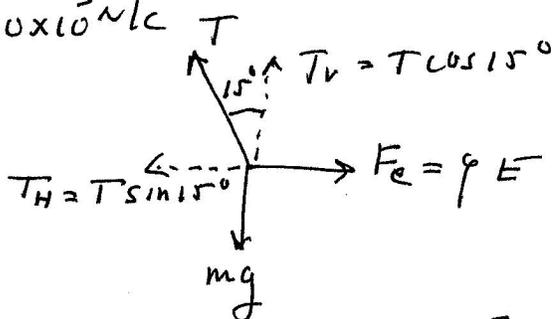
$E = \frac{5.20 \times 10^{-6} \text{ C/m}^2}{2(8.85 \times 10^{-12})} = 2.94 \times 10^5 \text{ N/C}$

15.52



$m = 2.00 \text{ gm}$

$E = 1.0 \times 10^3 \text{ N/C}$



$\Sigma F_H = F_e - T_H = 0 \rightarrow T \sin 15^\circ = F_e \quad \therefore \tan 15^\circ = \frac{F_e}{mg}$
 $\Sigma F_V = T_V - mg = 0 \rightarrow T \cos 15^\circ = mg$

$\tan 15^\circ = \frac{qE}{mg} \rightarrow q = \frac{mg \tan 15^\circ}{E}$

$q = \frac{(2 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2) \tan 15^\circ}{(1 \times 10^3 \text{ N/C})} = 5.25 \times 10^{-6} \text{ C} = 5.25 \mu\text{C}$