

Ch.24 - Wave Optics

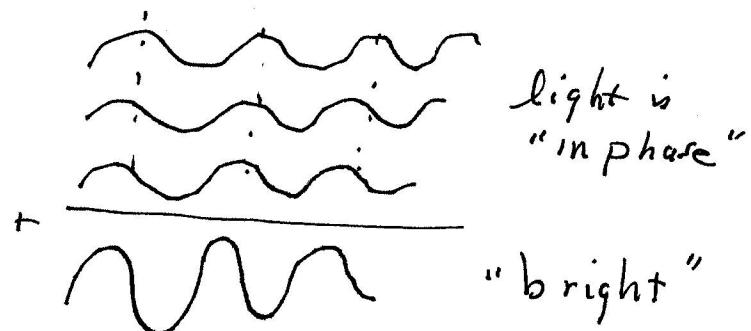
24.1

- 1) interference - adding of waves
- 2) diffraction - bending of light at a barrier.

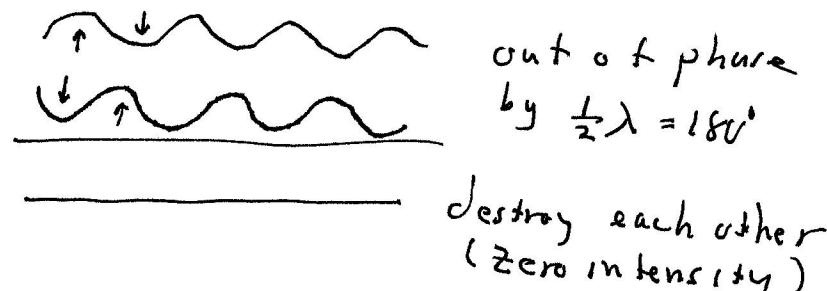
Interference:

1) totally constructive:

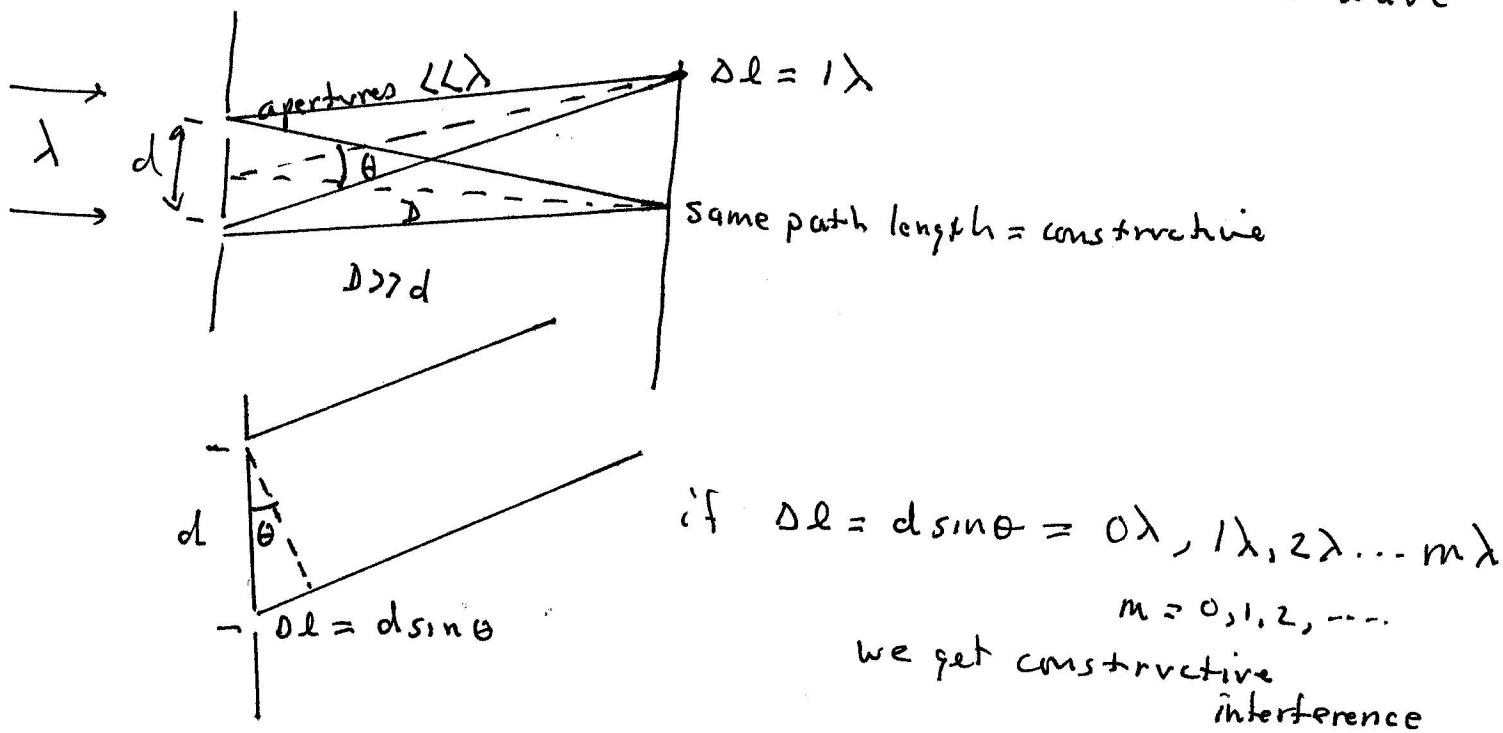
light is coherent +
mono chromatic



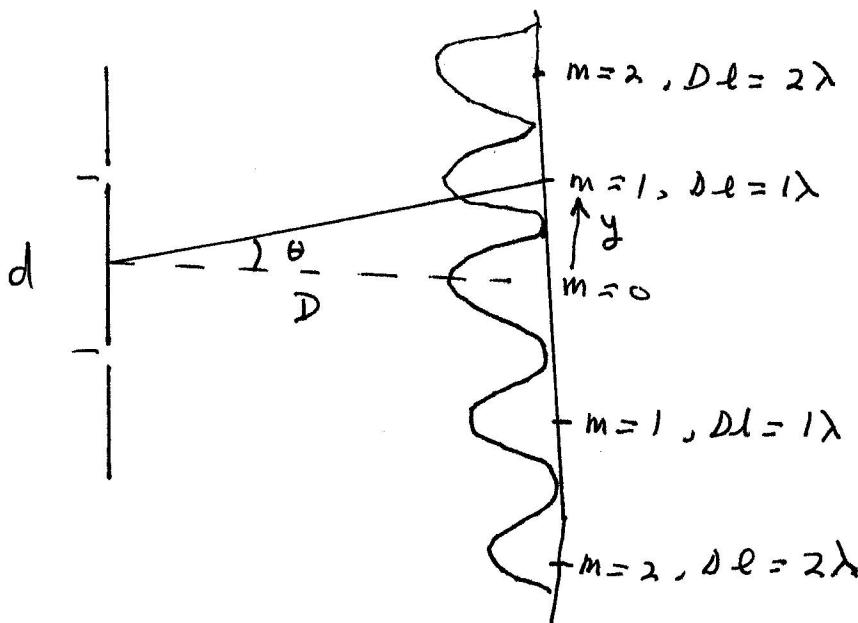
2) totally destructive



1801 - Young's Double slit experiment - demonstrated very well that light acts as a wave



24.2



$$1) \tan \theta = \frac{y}{D}$$

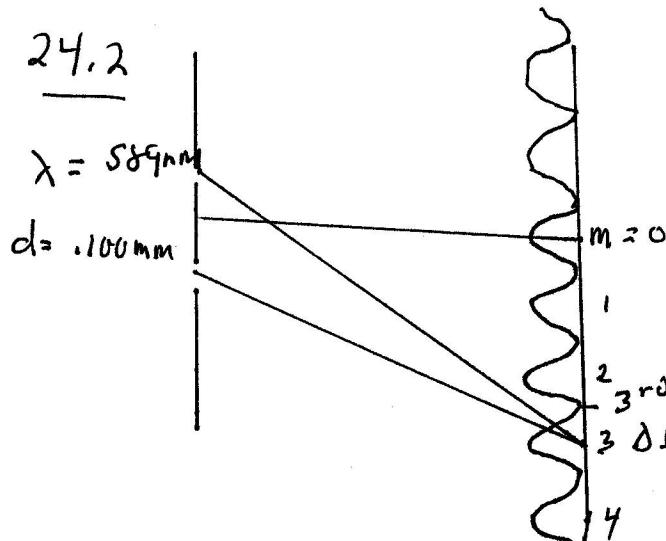
$$2) \text{for bright fringes} \\ d \sin \theta = m\lambda \\ m = 0, 1, 2, \dots$$

3) for dark bands,
the waves must be
different in path
length by $\frac{1}{2}\lambda$

$$d \sin \theta = (m + \frac{1}{2})\lambda$$

Note, bright fringes are
equally spaced and have "equal" intensity
= destructive

24.2



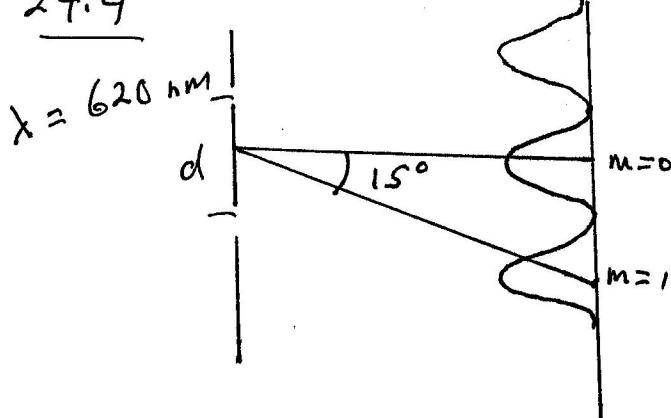
a) 3rd bright fringe, $m = 3$

$$\Delta l = 3\lambda = 3(589 \text{ nm}) = 1.77 \times 10^{-3} \text{ m} \\ = 1.77 \mu\text{m}$$

b) 3rd dark band $(\frac{1}{2}, \frac{3}{2}, \frac{5}{2})$

$$\Delta l = 2.5\lambda = 2.5(589 \text{ nm}) \\ = 1.47 \times 10^{-3} \text{ m} = 1.47 \mu\text{m}$$

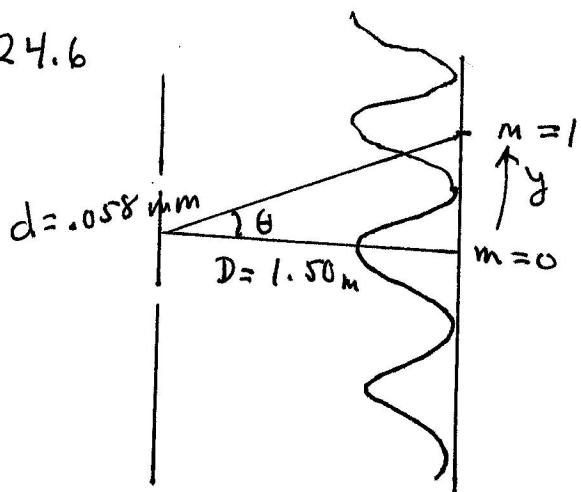
24.4



$$d \sin \theta = (1)\lambda$$

$$d = \frac{\lambda}{\sin \theta} = \frac{620 \times 10^{-9} \text{ m}}{\sin 15^\circ} = 2.40 \times 10^{-6} \text{ m} \\ = 2.40 \mu\text{m}$$

24.6

a) red, $\lambda = 588 \text{ nm}$

$$\alpha \sin \theta = \lambda$$

$$\sin \theta = \frac{\lambda}{d} = \frac{588 \times 10^{-9}}{0.58 \times 10^{-3}} = .0101, \theta = 0.58^\circ$$

$$\tan \theta = \frac{y}{D}, y = D \tan \theta$$

$$= 150 \text{ cm} \tan 0.58^\circ \\ = \boxed{1.52 \text{ cm}}$$

b) for blue, $\lambda = 412 \text{ nm}$

$$\alpha \sin \theta_2 = 2\lambda$$

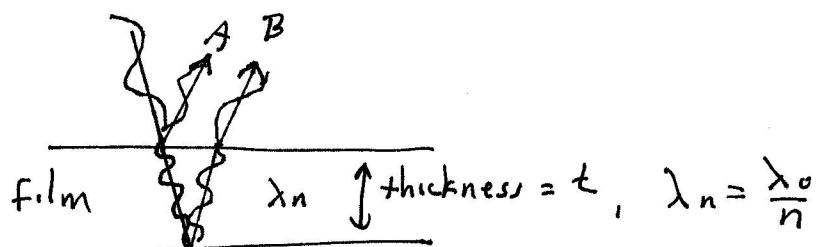
$$\sin \theta_2 = \frac{2(412 \times 10^{-9})}{0.58 \times 10^{-3}} = .0142, \theta_2 = 0.814^\circ$$

$$\sin \theta_4 = \frac{4(412 \times 10^{-9})}{0.58 \times 10^{-3}} = .0284, \theta_4 = 1.63^\circ$$

$$y_2 = 150 \text{ cm} \tan 0.814^\circ = 2.13 \text{ cm}$$

$$y_4 = 150 \text{ cm} \tan 1.63^\circ = 4.26 \text{ cm}$$

$$\Delta y = y_4 - y_2 = 4.26 \text{ cm} - 2.13 \text{ cm} = \boxed{2.13 \text{ cm}}$$

Thin films

reflection by A off the top interferes with ray B which has a path length = $2t$ and a lower reflection. Rays A and B then interfere as they leave the top of surface.

If the two rays differ in phase by a "whole" number of wavelengths ($\Delta\varphi_{\text{total}} = 0\lambda, 1\lambda, 2\lambda \dots m\lambda$) then we have constructive interference: $\Delta\varphi_{\text{total}} = \text{"whole"}\lambda =$

if the two rays differ in phase by a "half" number of wavelength, we have destructive interference

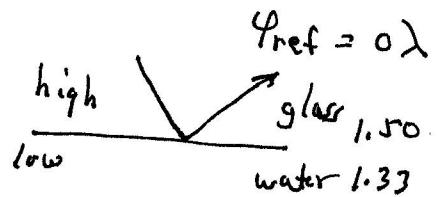
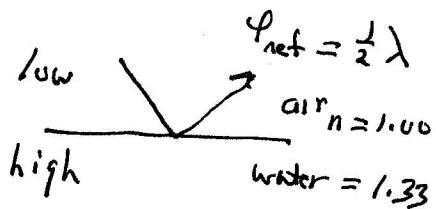
$$\Delta\varphi_{\text{total}} = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \dots = (m + \frac{1}{2})\lambda, m = 0, 1, 2 \dots$$

The total phase difference depends on two things

1) reflection

2) path length — we assume near perpendicular incidence
so the total path length = $2t$

For reflection:

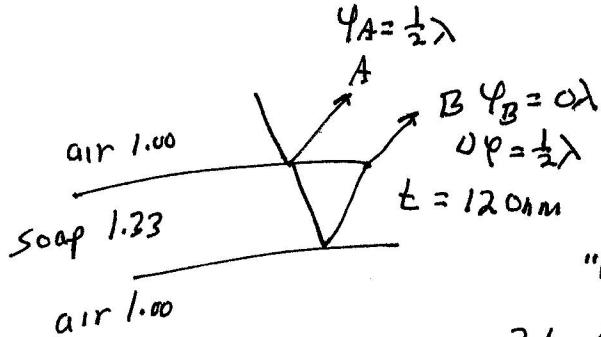


$$\Delta\varphi_{\text{total}} = \Delta\varphi_{\text{reflection}} + \Delta\varphi_{\text{path}}$$

"whole" $\lambda = \text{const. - bright}$
"half" $\lambda = \text{dest. - dark}$

$$\begin{aligned} 0\lambda \text{ or } \frac{1}{2}\lambda \\ M\lambda_n \quad (M + \frac{1}{2})\lambda_n \\ \frac{M\lambda_0}{n} \quad \frac{(M + \frac{1}{2})\lambda_0}{n} \end{aligned}$$

24.16



for constructive

$$\Delta\Phi_{\text{tot}} = \Delta\Phi_{\text{ref}} + N\Phi_{\text{path}}$$

$$\text{"whole"}\lambda = \frac{1}{2}\lambda + 2t$$

$$2t = \left(m + \frac{1}{2}\right) \frac{\lambda_0}{n}$$

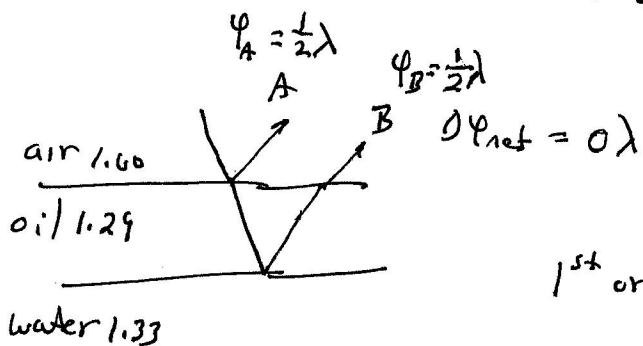
b) for $m = 0$, $\lambda_0 = \frac{2t}{m + \frac{1}{2}} = \frac{2(120 \text{ nm})(1.33)}{\frac{1}{2}} = 638 \text{ nm}$

for different thicknesses, we can still reflect 638 nm,
use different "m" values

c) $t = \frac{\left(m + \frac{1}{2}\right)\lambda_0}{2n}$

for $m = 1$: $t_1 = \frac{(1.5)(638 \text{ nm})}{2(1.33)} = 360 \text{ nm}$

for $m = 2$: $t_2 = \frac{(2.5)(638 \text{ nm})}{2(1.33)} = 600 \text{ nm}$

24.20constructive, $\lambda = 600 \text{ nm}$

$$\Delta\Phi_{\text{total}} = \Delta\Phi_{\text{ref}} + \Delta\Phi_{\text{path}}$$

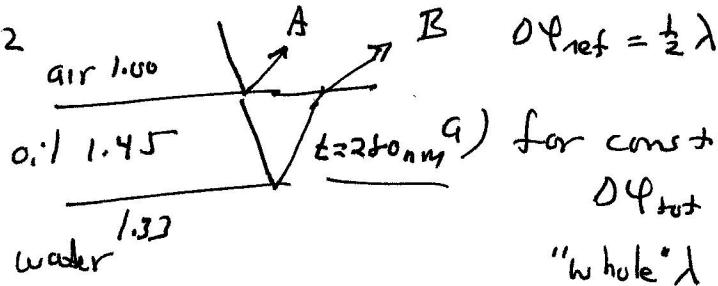
$$1^{\text{st}} \text{ order "whole"} = 0\lambda + 2t$$

$$\hookrightarrow \text{"whole"} = m\lambda_n$$

$$2t = m\lambda_0 = \frac{(1)\lambda_0}{n}$$

$$t = \frac{\lambda_0}{2n} = \frac{600 \text{ nm}}{2(1.29)} = 233 \text{ nm}$$

24.22



a) for constructive:

$$\Delta\varphi_{\text{tot}} = \Delta\varphi_{\text{ref}} + \Delta\varphi_{\text{path}}$$

$$\text{"whole"}\lambda = \frac{1}{2}\lambda + 2t \quad \text{"half"} = (m + \frac{1}{2})\frac{\lambda_0}{n}$$

$$\lambda_0 = \frac{2tn}{m + \frac{1}{2}} = \frac{2(280 \text{ nm})(1.45)}{m + \frac{1}{2}} = \frac{812 \text{ nm}}{m + \frac{1}{2}}$$

for $m=0$, $\lambda_0 = \frac{812 \text{ nm}}{\frac{1}{2}} = 1624 \text{ nm}$ (not visible)

$m=1$, $\lambda_0 = \frac{812 \text{ nm}}{1.5} = \boxed{541 \text{ nm} \quad \text{visible}}$

$m=2$, $\lambda_0 = \frac{812 \text{ nm}}{2.5} = 325 \text{ nm}$ not visible

b) for destructive:

$$\begin{aligned} \Delta\varphi_{\text{tot}} = \text{"half"}\lambda &= \Delta\varphi_{\text{ref}} + \Delta\varphi_{\text{path}} \\ &= \frac{1}{2}\lambda + 2t \end{aligned} \quad \text{"whole"} = m\frac{\lambda_0}{n}$$

$$\lambda_0 = \frac{2tn}{m} = \frac{812 \text{ nm}}{m}$$

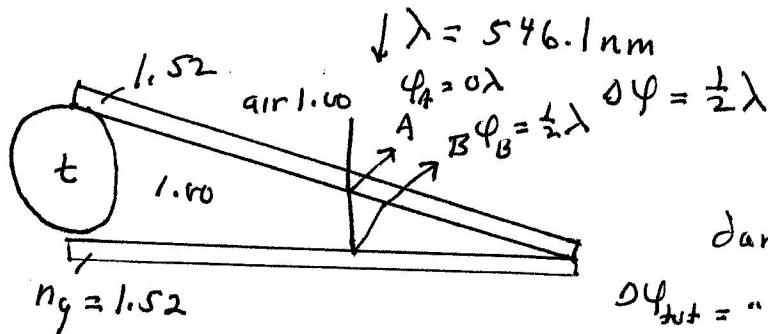
for $m=1$, $\lambda_0 = 812 \text{ nm}$ - not visible

$m=2$, $\lambda_0 = \frac{812 \text{ nm}}{2} = \boxed{406 \text{ nm} \quad \text{visible}}$

$m=3$, $\lambda_0 = \frac{812 \text{ nm}}{3} = 271 \text{ nm}$ not visible

24.24

$$t = 2.00 \mu\text{m} \\ = 2000 \text{ nm}$$



dark bands = destructive

$$\Delta\phi_{tot} = "half"\lambda = \Delta\phi_{ref} + \Delta\phi_{path} \\ = \frac{1}{2}\lambda + 2t \\ \hookrightarrow "whole"$$

$$f/m = \text{air}$$

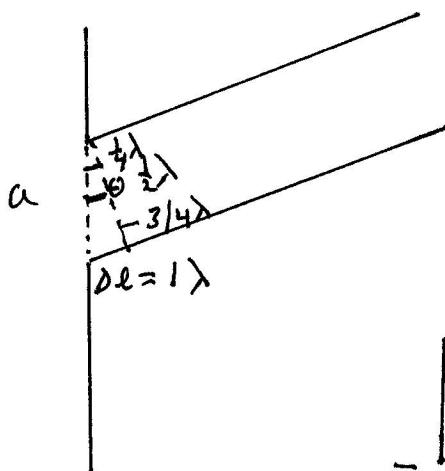
$$2t = m\lambda_n = m\frac{\lambda_0}{n}$$

$$m = \frac{2t}{\lambda_0} = \frac{2(2000 \text{ nm})(1.00)}{546.1 \text{ nm}} = 7.32$$

dark bands: $m = 0, 1, 2, 3, 4, 5, 6, 7 = \boxed{8 \text{ bands}}$

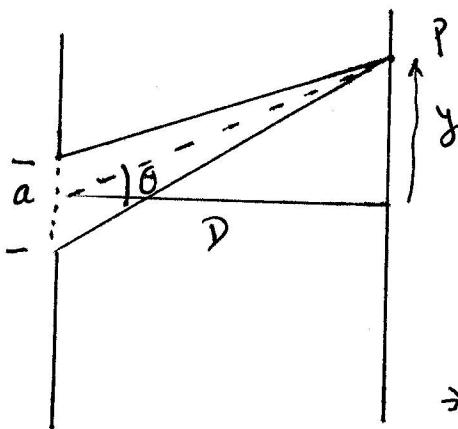
Single Aperture = diffraction

Consider an aperture where a plane wave strikes the slit such that the lower ray is 1λ longer than the top of the slit



for every ray in the upper $\frac{1}{2}\lambda$ there is a corresponding ray in the lower half that is $\frac{1}{2}\lambda$ longer

$\therefore a \sin \theta = m\lambda$ gives "dark" destructive interference patterns

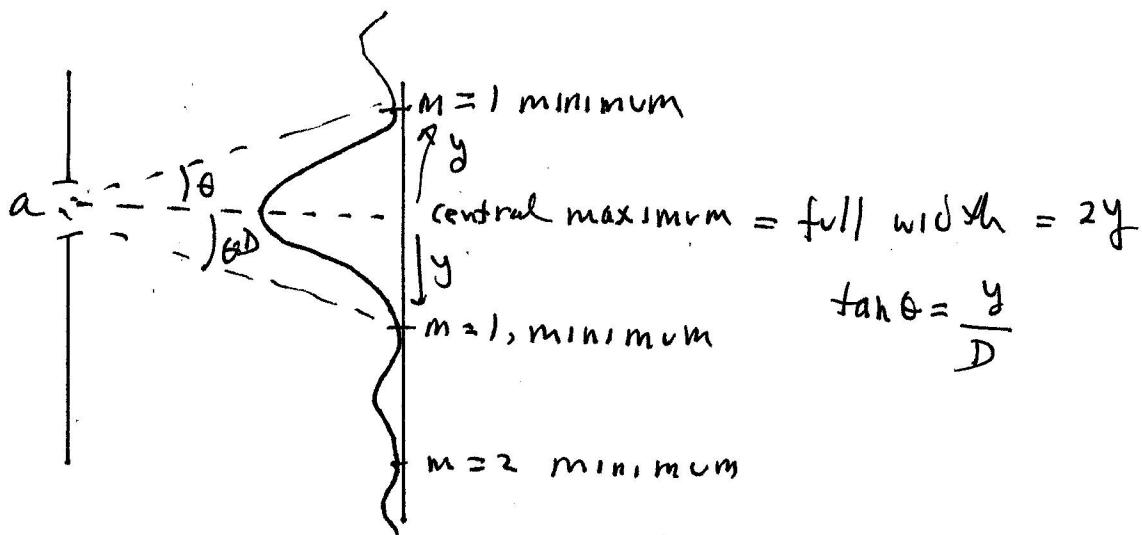


$$a \sin \theta = m\lambda = m\lambda_0 \text{ for minima} \\ m = 1, 2, 3, \dots$$

such that

$$\tan \theta = \frac{y}{D}$$

* remember, the point directly behind the aperture is a maximum ($m=0$)



* * Be careful for single slits: $a \sin \theta = m\lambda$

$$m = 1, 2, 3 \dots$$

= dark spots, minima

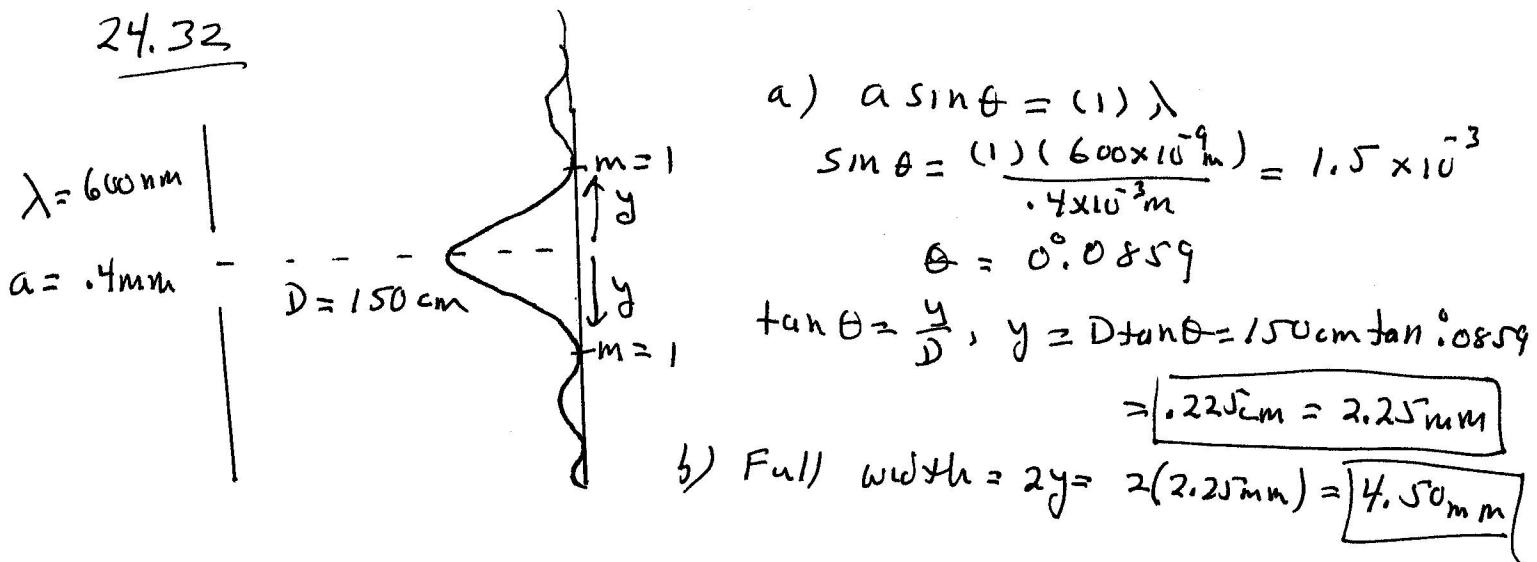
for double slits: $d \sin \theta = m\lambda$

$$M = 0, 1, 2, \dots$$

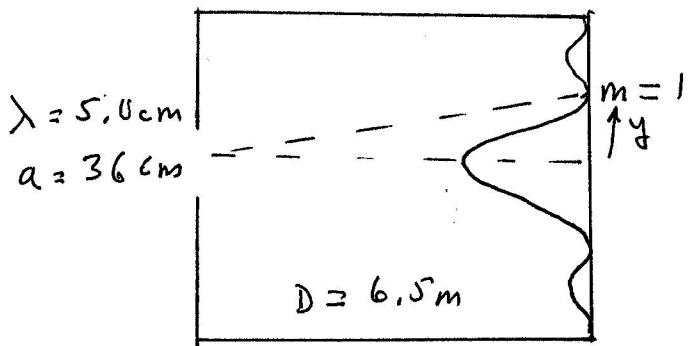
= bright fringes = bright spots

* the two m 's are separate and distinct - don't confuse "a" for "d" and "m" for interference with the different "m" for diffraction

24.32



24.9

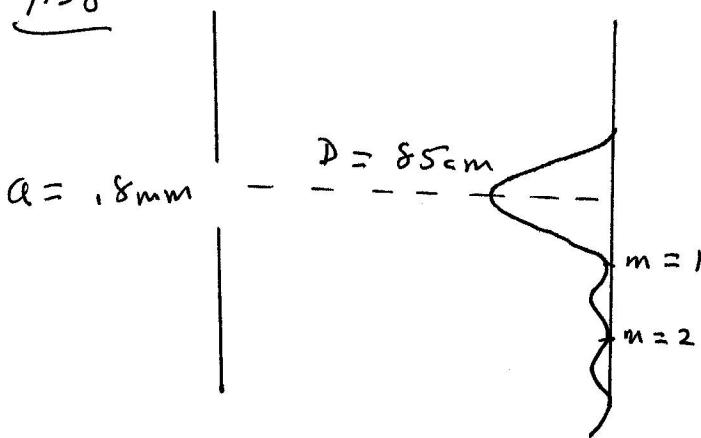
24.34

$$\text{1st min: } a \sin \theta = (1) \lambda$$

$$\sin \theta = \frac{5 \text{ cm}}{36 \text{ cm}} = .139, \theta = 7.98^\circ$$

$$\tan = \frac{y}{D} \rightarrow y = 650 \text{ cm} \tan 7.98^\circ$$

$$\boxed{y = 91.2 \text{ cm}}$$

24.38

$$\tan \theta = \frac{14 \text{ cm}}{85 \text{ cm}} = 1.65 \times 10^{-3}$$

$$\theta = 0^\circ.0944$$

$$a \sin \theta = 2\lambda$$

$$\lambda = \frac{a \sin \theta}{2} = \frac{(1.8 \times 10^{-3} \text{ m}) \sin (0.0944)}{2}$$

$$\lambda = 6.59 \times 10^{-7} \text{ m}$$

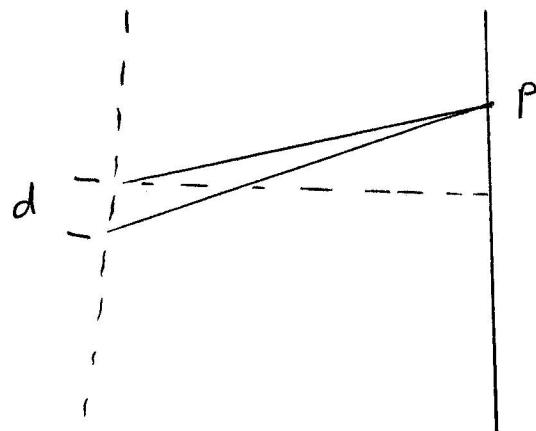
$$= 659 \times 10^9 \text{ nm} = \boxed{659 \text{ nm}}$$

Diffraction Gratings = many, many, slits, like

$$d = \frac{1}{\#} = \frac{\text{mm}}{600 \text{ lines}} = \frac{10^{-3} \text{ m}}{600 \text{ lines}} = 1.67 \times 10^{-6} \text{ m}$$

$$= 1.670 \times 10^{-9} \text{ m}$$

$$\underline{d = 1670 \text{ nm}}$$



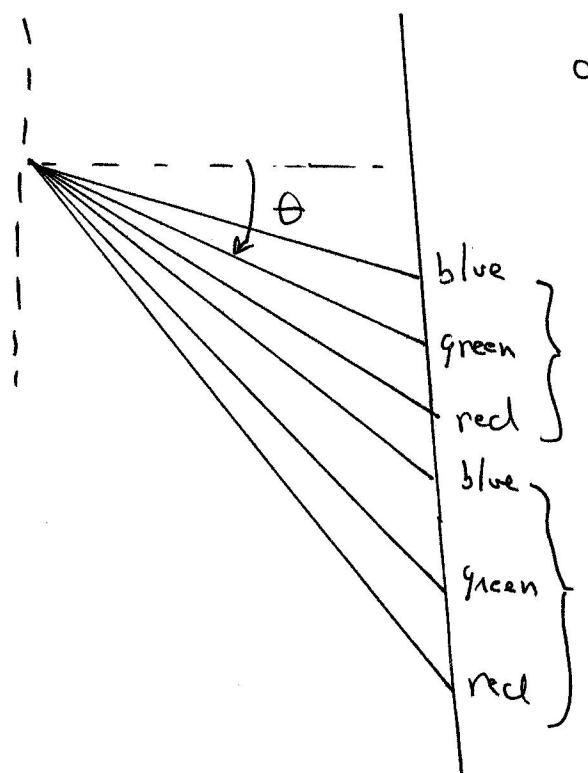
still we get constructive interference between the slits, like Young's double slit

$d \sin \theta = \Delta l = m\lambda \Rightarrow$ constructive interference bright

$\boxed{d \sin \theta = m\lambda}$ grating equation

but the maxima become much "sharper"

And the gratings act similar to prisms in breaking up the light
for 1st order



$d \sin \theta = \lambda$, as λ increases
(blue to red)
the angle θ increases

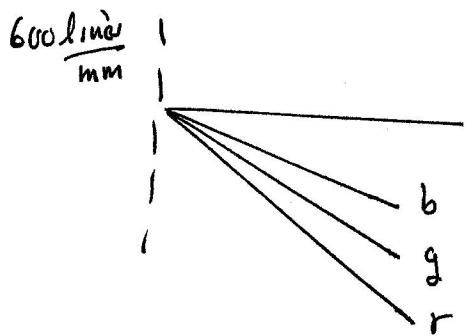
1st order
 $m=1$

2nd order
 $m=2$

maximum angle
 $= 90^\circ$

24.40

(visible light: 400nm - 700nm)



$$d = \frac{10^{-3} \text{ m}}{600 \text{ lines}} = 1.67 \times 10^{-6} \text{ m} = 1670 \text{ nm}$$

a) if you can see the red edge, you can see everything else \therefore test for the red edge

$$ds \sin \theta = m \lambda$$

\downarrow
 90° \downarrow
700 nm

$$m = \frac{ds \sin 90}{\lambda} = \frac{1670 \text{ nm} \sin 90}{700 \text{ nm}} = 2.39 \text{ + do not round up!}$$

\therefore [there are two orders on each side of central maximum]

b) for first order, $m = 1$

$$\sin \theta_b = \frac{(1)(400 \text{ nm})}{1670 \text{ nm}} = .24, \theta_b = 13.9^\circ$$

$$\sin \theta_r = \frac{(1)(700 \text{ nm})}{1670 \text{ nm}} = .419, \theta_r = 24.8^\circ$$

$$\Delta \theta = 10.9^\circ$$

24.43

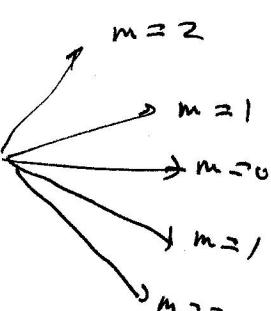
$$d = 1.30 \text{ cm}$$

$$f = 37.2 \text{ kHz}$$

$$v = 343 \text{ m/s}$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{37.2 \times 10^3 / \text{s}} = 9.22 \times 10^{-3} \text{ m} = 9.22 \text{ mm} = .922 \text{ cm}$$



$$ds \sin \theta = m \lambda$$

to find the number of maxima on each side, solve for m for $0 \rightarrow 90^\circ$

$$m = \frac{ds \sin 90}{\lambda} = \frac{1.3 \text{ cm} (1)}{.922 \text{ cm}} = 1.41$$

\Rightarrow one maximum on each side of central maximum:

$$m = 0, \pm 1 \Rightarrow \boxed{\text{total} = 3}$$

$$\sin \theta = \frac{(1)\lambda}{d} = \frac{(1) .922 \text{ cm}}{1.30 \text{ cm}} = .709$$

so, $\boxed{\text{angles are } 0^\circ, \pm 45.2^\circ}$

$$\theta = 45.2^\circ$$

$$\underline{24.44} \quad 2000 \frac{\text{lines}}{\text{cm}} \rightarrow d = \frac{10^{-2} \text{ m}}{2000 \text{ lines}} = 5.0 \times 10^{-6} \text{ m} = 5000 \text{ nm}$$

$$\lambda = 640 \text{ nm}, \quad d \sin \theta_1 = (1) \lambda, \quad \sin \theta_1 = \frac{\lambda}{d} = \frac{640 \text{ nm}}{5000 \text{ nm}} = .128$$

$$\boxed{\theta_1 = 7.35^\circ}$$

$$\underline{24.46} \quad v = \text{violet} = 400 \text{ nm}$$

$$r = \text{red} = 700 \text{ nm}$$

to overlap

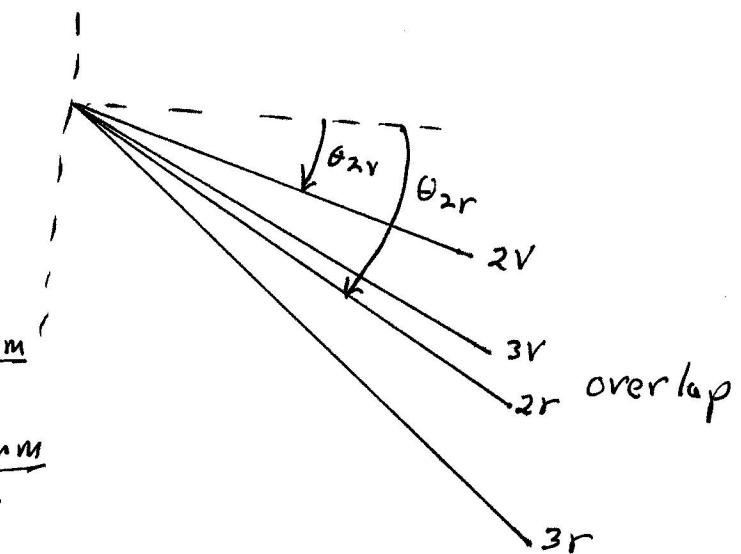
$$\theta_{3v} < \theta_{2r}$$

$$\sin \theta_{3v} = \frac{3(400 \text{ nm})}{d} = \frac{1200 \text{ nm}}{d}$$

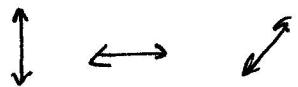
$$\sin \theta_{2r} = \frac{2(700 \text{ nm})}{d} = \frac{1400 \text{ nm}}{d}$$

$$\text{Since } \frac{1200 \text{ nm}}{d} < \frac{1400 \text{ nm}}{d}$$

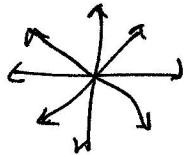
$$\theta_{3v} < \theta_{2r}$$



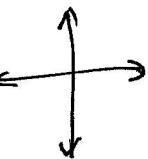
Plane Polarization = electric field limited to one plane



Unpolarized light is randomly distributed



characterized by

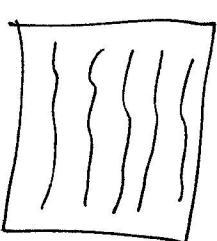
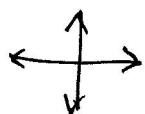


Each light burst produced is plane polarized, but when you sum all the various bursts, the combination is unpolarized.

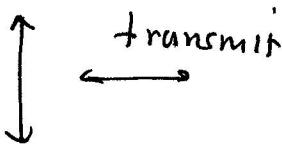
To convert unpolarized into plane polarized, there are 4 ways to do it.

- 1) Polaroids
- 2) scattering
- 3) reflection
- 4) birefringent crystals

1) Polaroids



absorb



transmit

We say



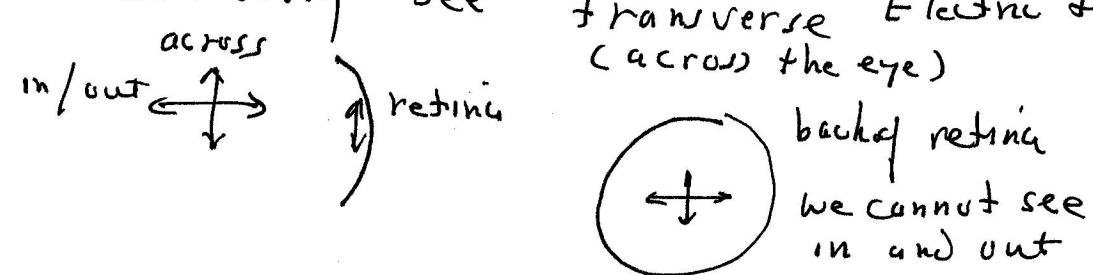
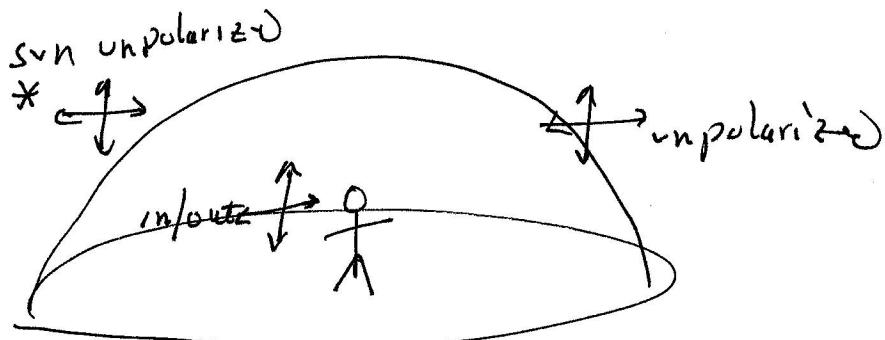
this is the transmission axis

long organic molecules
absorb components
of electric field
parallel to their
lengths and

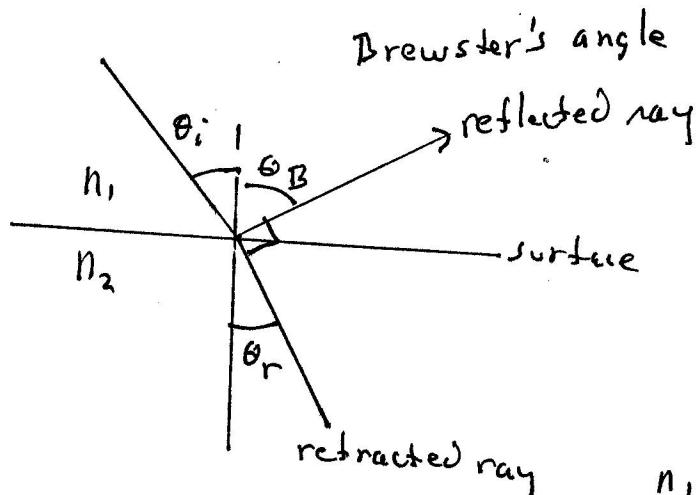
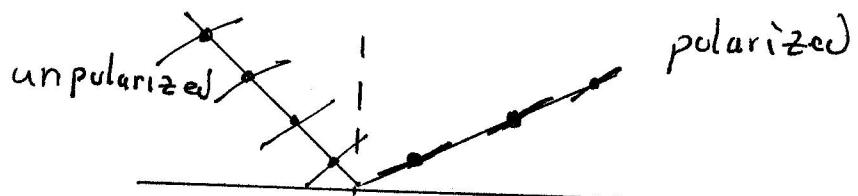
transmit

Perpendicular components

2) We can only "see" transverse Electric fields
 across the eye)

3) reflection



when light reflected
 is fully polarized
 $\theta_{\text{reflect}} + \theta_{\text{refract}} = 90^\circ$

from Snell's law

$$n_1 \sin \theta_B = n_2 \sin \theta_r$$

$$n_1 \sin \theta_B = n_2 \underbrace{\sin (90 - \theta_B)}_{\cos \theta_B}$$

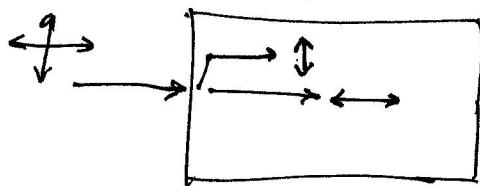
$$\frac{\sin \theta_B}{\cos \theta_B} = \frac{n_2}{n_1}$$

$$\tan \theta_B = \frac{n_2}{n_1} \quad \text{or if } n_1 = \text{air}$$

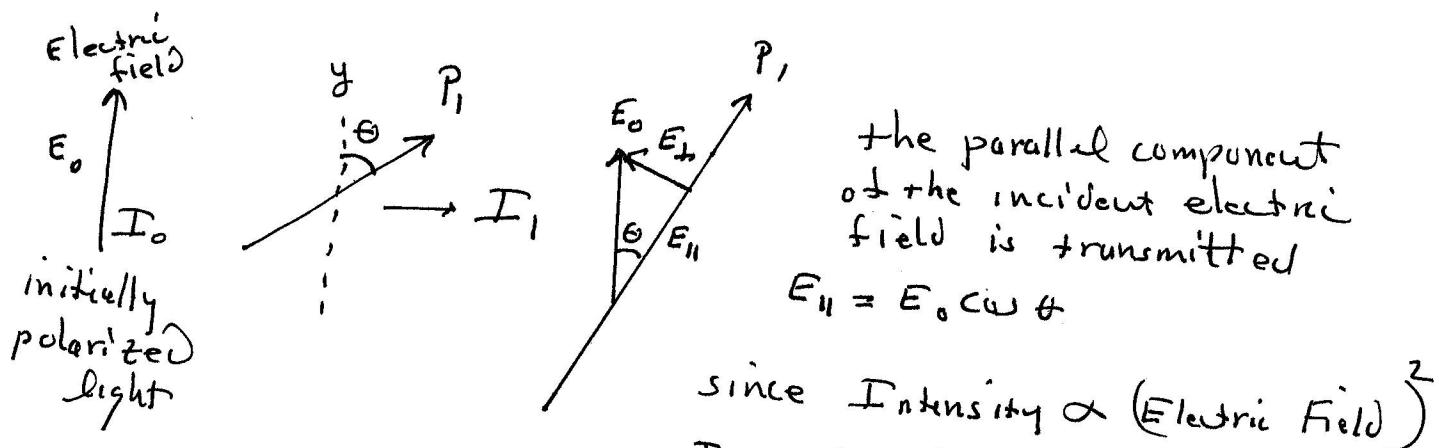
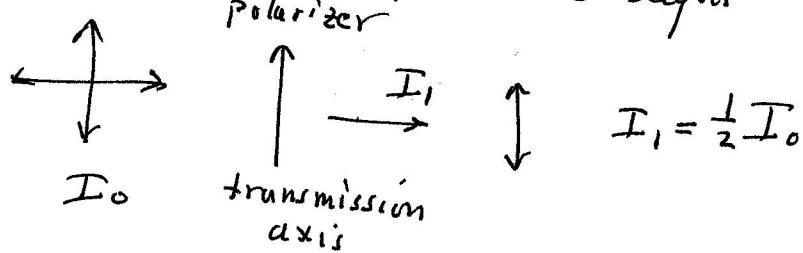
$$\boxed{\tan \theta_B = n}$$

4) birefringent crystals = calcite

↳ the crystal splits the incoming unpolarized light into two perpendicularly polarized components: the ordinary and the extra-ordinary
 the "ordinary" behaves like Snell's law
 the "extra-ordinary" has a different index of refraction and the light travels at a different speed.



Polaroids: ideal polaroids only absorb $\frac{1}{2}$ of incident unpolarized light



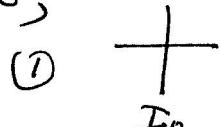
since Intensity $\propto (\text{Electric Field})^2$

$$\frac{I_1}{I_0} = \frac{(E_{\parallel})^2}{(E_0)^2} = \frac{E_0^2 \cos^2 \theta}{E_0^2} = \cos^2 \theta$$

$I_1 = I_0 \cos^2 \theta$

Law of Malus

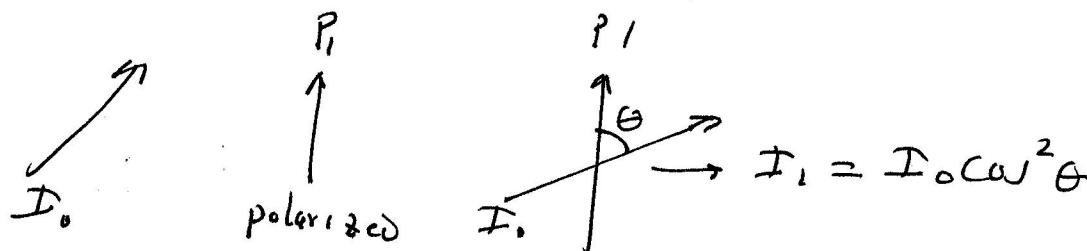
I_0

(1) 

on any polarizer \Rightarrow yields $I_1 = \frac{1}{2} I_0$

in the direction of the transmission direction

(2) any polarized light on a polarizer



24.53

I_0

P_1

$I_1 = \frac{1}{2} I_0$

P_1

$P_2 = \text{analyzer}$

$I_2 = I_1 \cos^2 35^\circ$

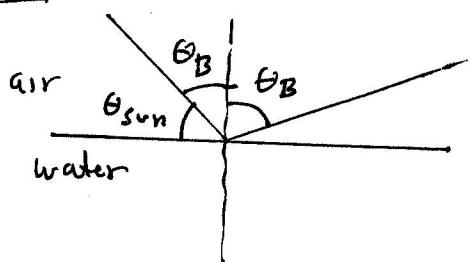
$= (\frac{1}{2} I_0) \cos^2 35^\circ = I_0 (.5) \cos^2 35^\circ$

$= .336 I_0$

$\boxed{\frac{I_2}{I_0} = .336 \text{ transmitted}}$

b) $.5 I_0$ is transmitted through P_1 to analyzer P_2
if $.336 I_0$ is transmitted by P_2 , then

$$(.5 - .336) I_0 \text{ is absorbed} = \boxed{.164 \text{ is absorbed}}$$

24.54

$$\tan \theta_B = \frac{n_w}{n_{air}} = \frac{4/3}{1} = 4/3$$

$$\theta_B = 53.1^\circ$$

$$\therefore \theta_{sun} \text{ to horizon} = 90 - \theta_B = \boxed{36.9^\circ}$$

24.60

a) \downarrow
 I_0

$$\begin{array}{c} P_1 \\ \uparrow \\ I_1 = I_0 \\ = I_0 \cos^2 0^\circ \end{array}$$

$$\begin{array}{c} 45^\circ \\ \nearrow \\ P_2 \\ \longrightarrow \\ I_2 \end{array}$$

$$\begin{aligned} I_2 &= I_1 \cos^2 45^\circ \\ &= I_0 \cos^2 45^\circ = \boxed{.50 I_0} \end{aligned}$$

b) for $I_2 = \frac{1}{3} I_0 = I_0 \cos^2 \theta$

$$\cos^2 \theta = \frac{1}{3} \rightarrow \cos \theta = \sqrt{\frac{1}{3}} = .577$$

$$\cos \theta = .577 \rightarrow \boxed{\theta = 54.7^\circ}$$