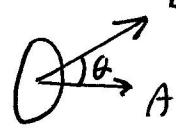
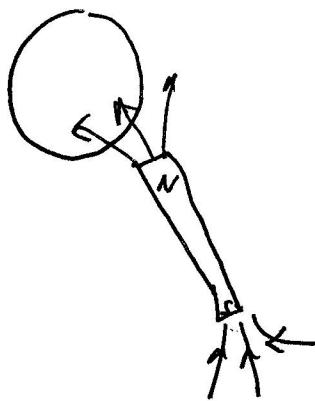


Ch.20: Faraday's Law + Induction

20.1

Faraday's Law \rightarrow induced voltages & currents

Magnetic flux $= \Phi_B =$ magnetic field lines passing through an area.



$$\Phi_B = B_{\perp} A = B A \cos \theta$$

$$= BA \cos \theta$$

$$= Tm^2$$

if the magnetic flux changes, an induced voltage or current appears

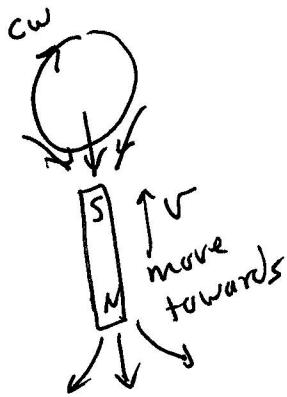
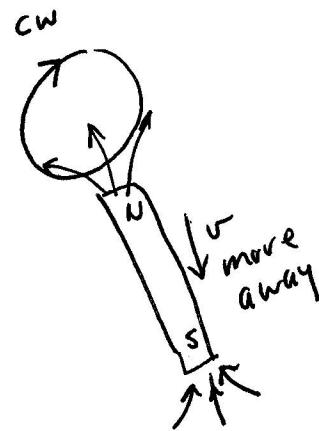
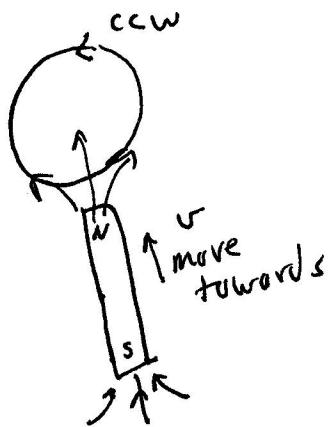
changing flux means changing magnetic field and/or changing area.

$$\Phi_B = BA \cos \theta$$

$$\frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta (BA \cos \theta)}{\Delta t} \Rightarrow \mathcal{E}, \quad i = \frac{\mathcal{E}}{R}$$

a) Faraday's Law: A changing magnetic flux creates an induced emf and current

b) Lenz's Law: The direction of the induced voltage or current is such that its effect opposes or counter-acts the change that created it in the first place.



To determine the direction of the induced voltage or current, there are two questions that must be answered:

- 1) How is the flux changing \rightarrow increasing or decreasing
- and 2) which way does the flux point? up - down
in - out
left - right ... ?

The direction of the induced B field (from the induced current) needs to oppose the change

if change = "increasing" $\rightarrow B_{\text{ind}}$ must point opposite to reduce

if change = "decreasing" $\rightarrow B_{\text{ind}}$ must point parallel to build it back up.

In general:

$$\boxed{\varepsilon = -N \frac{d\Phi_B}{dt}}$$

↓
Lenz's law
(direction)

Faraday's law

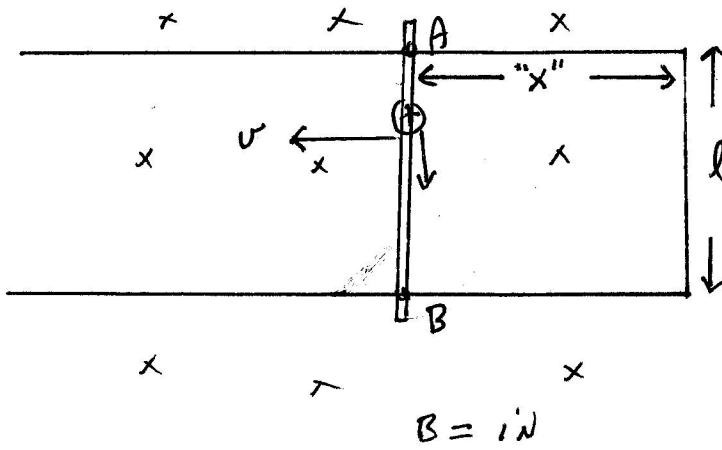
$N = \# \text{ turns (if coil)}$

$$\Phi_B = BA \cos\theta$$

Mechanical example → "generator"

two long ^{parallel} conducting rods, connected at one end with a sliding conducting rod traveling along the rails in a uniform magnetic field.

The sliding rod moves to the left with speed v
(due to external force)



emf = two different ways

$$\Delta V_{AB} = \frac{\text{Work}}{\text{charge}} = \frac{F_m \cdot l}{q}$$

$$= \frac{qvB \cdot l}{q} = vBl \quad (\text{volts})$$

as long as $v = \text{const.}$

then $\varepsilon = \text{const.}$

this emf converts "mechanical" energy into electrical energy

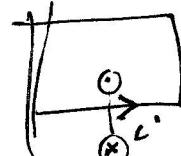
b) by Faraday's law: $\Phi_B = BA \cos\theta = B(xl) \cos 0^\circ$, $N=1$

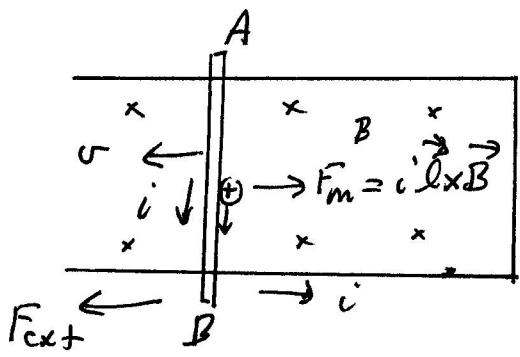
$$\varepsilon = \frac{d\Phi}{dt} = B \cdot \frac{dx}{dt} \cdot l = \underline{\underline{Bvl}} !$$

changing flux: increasing, in [Area is increasing] ↗

∴ $B_{\text{induced}} \Rightarrow$ point opposite = out ∴ $i =$ ↗

→ Same answer





1. Just because the current runs from A to B or "out of B" does not make $V_B > V_A$!

The magnetic force pushes the charges around \rightarrow doing work and $\Delta V = \frac{W}{q}$!

2. Note: since the current flows from A to B it interacts with the \vec{B} field with a force $\vec{F}_m = i\vec{l} \times \vec{B}$ to the right, that opposes the velocity, slowing down the rod. The F_m acts to "oppose" the change creating it.

So to keep the current flowing, the emf needs an external force (F_{ext}) pointing to the left to keep the rod moving \rightarrow "you can't get something for nothing!" - 2nd Law of Thermodynamics"

To maintain constant velocity (constant current) the external force must equal the retarding magnetic force

$$F_{ext} = F_m = ilB = \frac{\epsilon lB}{R}$$

OR output power = input power (at a minimum)

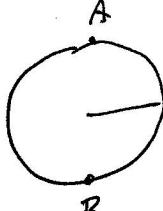
$$i^2 R = \frac{\epsilon^2}{R} = i\epsilon = F_{ext} \cdot v \quad (\text{watts})$$

$$\underline{20.8} \quad \Delta B = 0 \rightarrow 1.5T, \Delta t = 120ms = .12s$$



$$\epsilon = \frac{\Delta(BA)}{\Delta t} = \frac{\Delta B}{\Delta t} \cdot A$$

$$\epsilon = \left(\frac{1.5T - 0}{.12s} \right) \pi (1.6 \times 10^{-3}m)^2 = \frac{1.01 \times 10^{-4}V}{= \boxed{.10mV}}$$

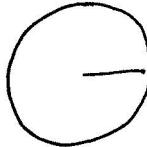
$$\underline{20.10}$$


$$B = .15T$$

loop is flattened in $\Delta t = .20s$

$$\epsilon = \frac{\Delta(BA)}{\Delta t} = B \frac{\Delta A}{\Delta t} = (.15T) \left(\frac{\pi r^2}{\Delta t} \right)$$

$$= (.15T) \left(\frac{\pi (12cm)^2}{.20s} \right) = \boxed{3.39 \times 10^{-2}V}$$

$$\underline{20.12}$$


$$r = 12cm \quad \frac{\Delta B}{\Delta t} = .05T/s$$

$$\epsilon = \frac{\Delta(BA)}{\Delta t} = \frac{\Delta B}{\Delta t} \cdot A = (.05T/s) \pi (12cm)^2 = \boxed{2.26 \times 10^{-3}V}$$

2.26 mV

$$\underline{20.20}$$



$$N = 200 \text{ turns}$$

$$R = 5.0cm$$

$$B_i = 1.1T \text{ out } \oplus$$

$$B_f = 1.1T \text{ in } \ominus$$

$$\Delta t = .10s$$

$$A = 100 \text{ cm}^2$$

$$\epsilon = N \frac{\Delta(BA)}{\Delta t} = N \frac{\Delta B}{\Delta t} A$$

$$\epsilon = 200 \left(\frac{1.1T - (-1.1T)}{.1s} \right) 100 \times 10^{-4} \text{ m}^2$$

$$= 44V$$

$$i = \frac{\epsilon}{R} = \frac{44V}{5\Omega} = \boxed{8.8A}$$

20.24

$$L = 2.0 \text{ m} \quad v = 15 \text{ m/s}$$

$$B_v = 40 \mu\text{T} \quad \mathcal{E} = v L B = (15 \text{ m/s})(2 \text{ m})(40 \times 10^{-6}) = \boxed{1.20 \times 10^{-3} \text{ V}}$$

$$1.20 \text{ mV}$$

20.30

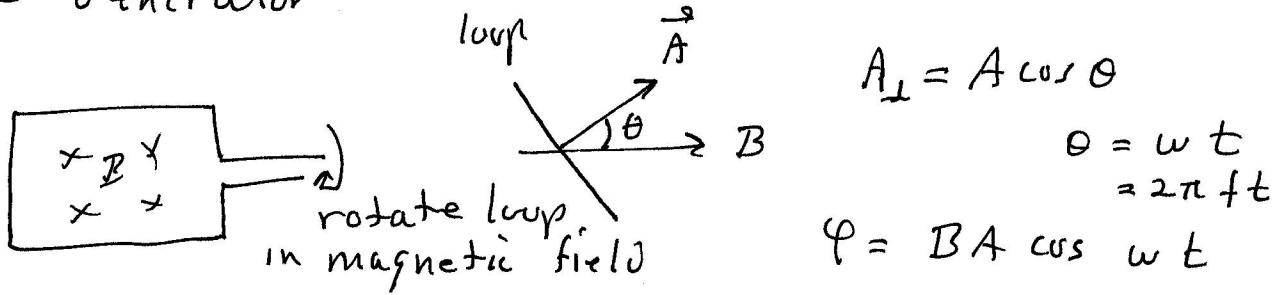
$$R = 6 \Omega \quad B = 1 \text{ T} = 2.50 \text{ T}$$

$$\ell = 1.2 \text{ m} \quad i = .50 \text{ A}$$

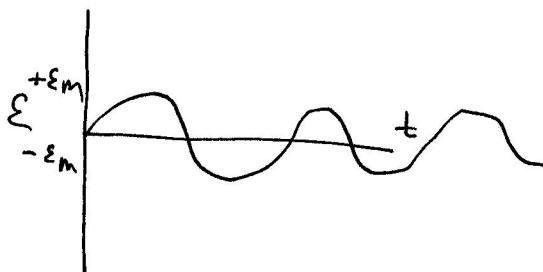
$$V = i R = (.50 \text{ A})(6 \Omega) = 3.0 \text{ V}$$

$$\mathcal{E} = 3 \text{ V} = v \ell B, \quad v = \frac{3 \text{ V}}{(1.2 \text{ m})(2.5 \text{ T})} = \boxed{1.0 \text{ m/s}}$$

AC Generator



$$\frac{d\varphi}{dt} = BA\omega \sin \omega t$$

for N turns

$$\mathcal{E} = \underbrace{NBA\omega}_{\mathcal{E}_m} \sin \omega t$$

Self Inductance, $L = \text{Henry's}$

 = inductor

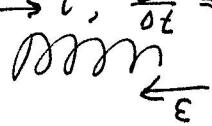
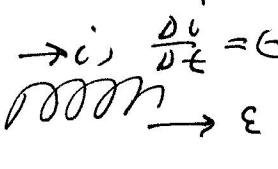
A coil can interfere with itself

$$L = \frac{N\varphi}{i}, N\varphi = \text{flux linkage}$$

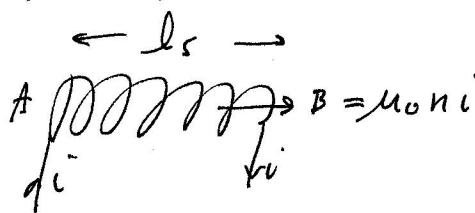
$$\hookrightarrow N\varphi = L_i$$

$$N \frac{d\varphi}{dt} = L \frac{di}{dt} = \varepsilon \quad \text{and direction of emf is given by Lenz's Law}$$

$$\rightarrow i, \frac{di}{dt} = \Theta \quad \rightarrow i, \frac{di}{dt} = \ominus$$

for a solenoid



$$L = \frac{N\varphi}{i} = \frac{(Nl_s)(BA)}{i}$$

$$L = \frac{(Nl_s)(\mu_0 n^2 A)}{i} = \boxed{\mu_0 n^2 l_s A} = L$$

20.38

solenoid



$$n = \frac{N}{l} = \frac{580}{.36\text{m}} = 1.61 \times 10^3 \text{ turns/m}$$

$$L = \mu_0 n^2 l A = \mu_0 h^2 l \frac{\pi d^2}{4}$$

$$L = (4\pi \times 10^{-7})(1.61 \times 10^3)^2 (.36\text{m}) \pi \frac{(.08\text{m})^2}{4} = \boxed{5.89 \times 10^{-3} \text{ H}}$$

$$\varepsilon = L \frac{di}{dt} = (5.89 \times 10^{-3} \text{ H})(4\text{A/s}) = .024 \text{ V} = \boxed{24 \text{ mV}}$$

20.42

$$\varepsilon = L \frac{di}{dt} = \frac{N\varphi}{i} \frac{di}{dt} \rightarrow \varphi = \frac{\varepsilon i}{N \frac{di}{dt}}$$

$$\varepsilon = 24 \text{ mV}$$

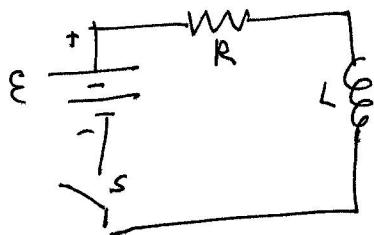
$$i = 4.0 \text{ A}$$

$$\frac{di}{dt} = 10 \text{ A/s}$$

$$N = 500 \text{ turns}$$

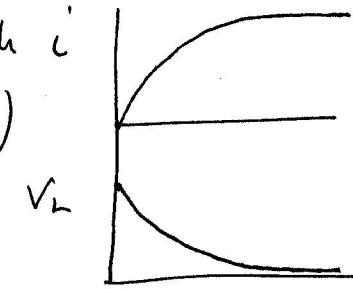
$$\varphi = \frac{(24 \times 10^{-3} \text{ V})(4.0 \text{ A})}{(500 \text{ turns})(10 \text{ A/s})}$$

$$\boxed{\varphi = 1.92 \times 10^{-5} \text{ Tm}^2 / \text{turn}}$$

L-R Series Circuit

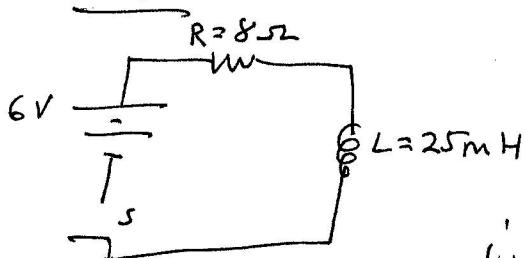
close switch i
 $i(t) = i_m (1 - e^{-t/\tau})$

$\tau = \frac{L}{R}$

 V_L

stored energy = $\frac{1}{2} L i^2$

20.46

switch closed at $t=0$:

a) find V_R at $t=0, t=1\tau$

at $t=0, i=0 \therefore V=iR=0$

b) at $t=1\tau, i = i_m (1 - e^{-t/\tau})$

$i_m = \frac{6V}{8\Omega} = .75A \quad i = .75A (1 - e^{-1}) = .474A$

$V_R = iR = (.474A)(8\Omega) = 3.79V$

$V_R + V_L = 6V, V_L = 6V - V_R$

so, at $t=0, V_L = 6V - 0 = 6V$

at $t=1\tau, V_L = 6V - 3.79V = 2.21V$

20.50 solenoid

$N = 300$ turns

a) $L = \mu_0 n^2 l A$

$r = 5.0$ cm

$n = \frac{300 \text{ turns}}{.2m} = 1.5 \times 10^3 \text{ turns/m}$

$l = 20$ cm

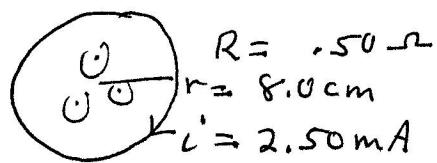
$L = \mu_0 n^2 l \pi r^2$

$$L = (4\pi \times 10^{-7})(1.5 \times 10^3)^2 (0.2m) \pi (0.05m)^2$$

$$= 4.44 \times 10^{-3} H$$

b) when $i = 0.50A$

$U = \frac{1}{2} L i^2 = \frac{1}{2} (4.44 \times 10^{-3} H) (0.5A)^2 = 5.55 \times 10^{-4} J$

20.54 $B = \text{out}$ 

- a) for a CW current $\rightarrow B_{\text{induced}} = iN$
 so by Lenz's law it must oppose the
 change, since iN is opposite to
 existing field, the existing out
 field must be increasing out

$$\mathcal{E} = iR = (2.50 \times 10^{-3}\text{A})(.5\text{m}) = 1.25 \times 10^{-3}\text{V} = \frac{\Delta(BA)}{\Delta t}$$

$$A = \pi r^2 = \pi (.05\text{m})^2 = 2.0 \times 10^{-2}\text{m}^2$$

$$\therefore \frac{\Delta B}{\Delta t} = \frac{\mathcal{E}}{A} = \frac{1.25 \times 10^{-3}\text{V}}{2 \times 10^{-2}\text{m}^2} = \boxed{.0625\text{T/s}} = \boxed{62.5\text{mT/s}}$$