

Ch. 16 - Electric Potential

From the work-energy relationship $W = \vec{F} \cdot \vec{d} = \Delta K$

$$\vec{F} = \text{conservative} + \text{non-conservative} = \vec{F}_c + \vec{F}_{nc}$$

$$W_{\text{net}} = (\vec{F}_c + \vec{F}_{nc}) \cdot \vec{d} = \Delta K = \vec{F}_c \cdot \vec{d} + \vec{F}_{nc} \cdot \vec{d} = W_c + W_{nc}$$

We define potential energy ^(U) only for conservative forces

i.e. gravity, elastic (springs)

as well as electric and magnetic forces

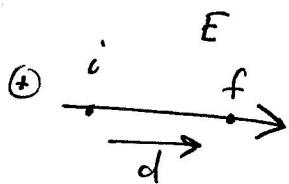
such that $\Delta U + \Delta K = 0 \Rightarrow \Delta U = -\Delta K$

$$\Delta U = -\Delta K = -W_c \quad (\text{for conservative forces})$$

We define electric potential $V = \frac{U_e}{q}$ J/C = volt

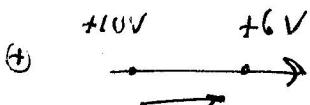
$$\text{or } \Delta V = \frac{\Delta U_e}{q} = -\frac{W_e}{q} \text{ Volts}$$

such that $\Delta U = q \Delta V \rightarrow \Delta K$.



$$W_c = \vec{F}_e \cdot \vec{d} = qEd$$

$$\Delta V = V_f - V_i = -\frac{W_e}{q} = -\frac{qEd}{q}$$



$$\Delta V = V_f - V_i = (+6V) - (+10V) = -4V$$

$$= -Ed$$

4 terms and concepts:

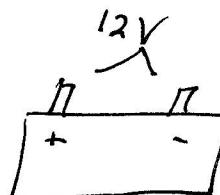
U_e = electric potential energy

ΔU_e = electric potential energy difference \rightarrow Joules

V = electric potential

ΔV = electric potential difference \rightarrow Volts

Measurements made with a voltmeter are "difference" measurements
i.e. Car battery = 12 volts



so what does it mean? $V_+ - V_- = 12$ volts

what is V_+ , what is V_- ?

V_A	V_B	ΔV
+12	0	12V
+6	-6	12V
112	100	12V
1012	1000	12V
10,012	10,000	12V

charge accelerated through potential difference

$$+ | \overrightarrow{\oplus} | - \quad \Delta K + \Delta U = 0$$

$$\Delta K + q \Delta V = 0$$

$$\Delta K = -q \Delta V$$

\hookrightarrow potential drop

$$= -q (-\text{value}) = (+) \times (-) = + = \Delta K$$

$q = \text{coulombs}$

gain KE

$$\Delta V = \text{volts} = \frac{F}{C}$$

$$\Delta K = (C)(V) = C \left(\frac{J}{C} \right) = \text{Joules}$$

$$\Delta \frac{1}{2} mv^2 = \text{Joules} = q \Delta V$$

OR

ΔK can be measured in "eV's"

$$q \Delta V = (\# \text{electrons}) \times (\text{eV in volts}) = \text{eV}$$

$$1 \text{eV} = (1.6 \times 10^{-19} \text{C})(1 \text{V}) = 1.6 \times 10^{-19} \text{J}$$

1 electron or singly ionized particle

$$1 \text{eV} = 1.60 \times 10^{-19} \text{J}$$

20 V

$$| A^+ | \rightarrow \Delta K = (1/e)(20 \text{V}) = \underline{\underline{20 \text{ eV}}}$$

$$20 \text{eV} = \Delta K = \Delta \left(\frac{1}{2} mv^2 \right)$$

↓
convert
to Joules

$$20 \text{eV} \times 1.6 \times 10^{-19} \text{J/eV} \rightarrow \text{Joules} = \Delta \left(\frac{1}{2} mv^2 \right)$$

16.6

$$\begin{aligned} \cdot q &= +40 \text{nC} \\ \frac{q}{E} &= 275 \text{ N/C} \\ A \quad d &= .18 \text{ m} \quad B \end{aligned}$$

$$\begin{aligned} a) F &= qE = (40 \times 10^{-6} \text{ C})(275 \frac{\text{N}}{\text{C}}) = \boxed{1.10 \times 10^{-2} \text{ N}} \\ b) W_e &= Ed = (1.10 \times 10^{-2} \text{ N})(.18 \text{ m}) = \boxed{1.98 \times 10^{-3} \text{ J}} \\ c) \Delta U_e &= -W_e = \boxed{-1.98 \times 10^{-3} \text{ J}} \\ d) \Delta V &= \frac{U_e}{F} = -Ed = -(275 \frac{\text{V}}{\text{m}})(.18 \text{ m}) = \boxed{-49.5 \text{ V}} \end{aligned}$$

16.8 $\Delta U = -\Delta K = -(\cancel{k_f^0} - k_i) = +k_i = \frac{1}{2} m v_i^2$

a) to stop electron $\Delta U = -e \Delta V_e = \frac{1}{2} m_e v_i^2$

$$\Delta V_e = \frac{\frac{1}{2} m_e v_i^2}{-e} = + \frac{\frac{1}{2} (9.11 \times 10^{-31}) (2.85 \times 10^7 \text{ m/s})^2}{-1.6 \times 10^{-19} \text{ C}} = \boxed{-2.31 \times 10^3 \text{ volts}}$$

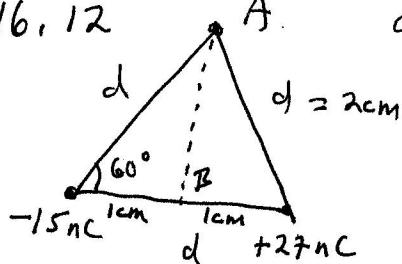
b) to stop proton: $e \Delta V_p = \frac{1}{2} m_p v_i^2$

$$\Delta V_p = \frac{\frac{1}{2} m_p v_i^2}{e}, \text{ since } m_p > m_e \text{ then } k_p > k_e \\ \therefore \Delta V_p > \Delta V_e$$

c) $\Delta V_p = \frac{\frac{1}{2} m_p v_i^2}{e}$

$$\Delta V_e = -\frac{\frac{1}{2} m_e v_i^2}{e} \quad \therefore \boxed{\frac{\Delta V_p}{\Delta V_e} = -\frac{m_p}{m_e}}$$

16.12



$$\begin{aligned} a) V_A &= \frac{kq_-}{d} + \frac{kq_+}{d} \\ &= \frac{(9 \times 10^9)(-15 \times 10^{-9})}{(0.2 \text{ m})} + \frac{(9 \times 10^9)(27 \times 10^{-9})}{(0.2 \text{ m})} \\ &= -6.75 \times 10^3 \text{ V} + 1.22 \times 10^4 \text{ V} \\ \boxed{V_A = +5.40 \times 10^3 \text{ V}} &= 5.40 \text{ kV} \end{aligned}$$

$$\begin{aligned} b) V_B &= \frac{(9 \times 10^9)(-15 \times 10^{-9})}{(0.1 \text{ m})} + \frac{(9 \times 10^9)(27 \times 10^{-9})}{(0.1 \text{ m})} \\ &= -1.35 \times 10^4 \text{ V} + 2.43 \times 10^4 \text{ V} \end{aligned}$$

$$V_B = \boxed{+1.08 \times 10^4 \text{ V}} = 10.8 \text{ kV}$$

16.4

16.20

16.5

$$\begin{array}{c} 4 \times 10^{-15} \text{ m} \\ \bullet \quad \bullet \\ q_2 = e \quad p \\ M_\alpha = 6.64 \times 10^{-27} \text{ kg} \quad m_p = 1.67 \times 10^{-27} \text{ kg} \end{array}$$

a) there are two unknowns
(V_α, V_p) so need two independent equations

b) conservation of i) energy
ii) linear momentum

$$\delta U = -DK \rightarrow U_f - U_i = -(K_f - K_i) \rightarrow f U_i = f K_f$$

$$U_i = q_p V_\alpha = \frac{k q_p q_\alpha}{r_i} = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} m_p V_p^2$$

such that $p_i = p_f$

$$\Theta = m_p V_p - M_\alpha V_\alpha \rightarrow m_p V_p = M_\alpha V_\alpha \rightarrow$$

$$V_p = \frac{M_\alpha}{m_p} V_\alpha$$

$$U_i = \frac{(8 \times 10^9)(1.6 \times 10^{-14})(2 \times 1.6 \times 10^{-14})}{4 \times 10^{-15} \text{ m}} = \frac{1}{2} (6.64 \times 10^{-27}) V_\alpha^2 + \frac{1}{2} (1.67 \times 10^{-27}) V_p^2$$

$$\begin{aligned} 1.15 \times 10^{-13} \text{ J} &= 3.32 \times 10^{-27} V_\alpha^2 + 8.35 \times 10^{-27} V_p^2 \\ &= 3.32 \times 10^{-27} V_\alpha^2 + 8.35 \times 10^{-27} \left(\frac{6.64 \times 10^{-27}}{1.67 \times 10^{-27}} \right)^2 V_\alpha^2 \\ &\quad + 1.32 \times 10^{-26} V_\alpha^2 \end{aligned}$$

$$1.15 \times 10^{-13} \text{ J} = 1.65 \times 10^{-26} V_\alpha^2$$

$$V_\alpha^2 = \frac{1.15 \times 10^{-13}}{1.65 \times 10^{-26}} = 6.96 \times 10^{12} \rightarrow \boxed{V_\alpha = \sqrt{6.96 \times 10^{12}} = 2.64 \times 10^6 \text{ m/s}}$$

$$V_p = \left(\frac{6.64 \times 10^{-27}}{1.67 \times 10^{-27}} \right) 2.64 \times 10^6 \text{ m/s} \rightarrow \boxed{1.05 \times 10^7 \text{ m/s} = V_p}$$

Capacitance / Capacitors
 ↴
 the Concept ↴
 the device —/—

The concept: to charge a conductor \rightarrow charge (Q) is added and electric potential increases: $V = \frac{kQ}{r}$

the relationship between charge and potential is $C = C = \frac{\partial Q}{\partial V}$ (for a sphere)



$$C = \frac{\partial Q}{\partial V} \quad \text{or} \quad C = \frac{Q}{V} = \text{Farads}$$

$C \rightarrow$ usually
MF or μF
 $\times 10^6$ $\times 10^{-12}$

for a sphere $V = \frac{kQ}{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

so, if $Q = CV$, then $\downarrow Q = (4\pi\epsilon_0 r) V$

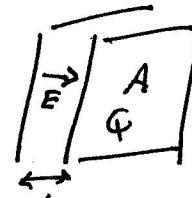
$$\downarrow C = 4\pi\epsilon_0 r$$

Capacitance of sphere

or for parallel plate capacitor:

$$\partial V = E d = \frac{\sigma}{\epsilon_0} d$$

$$\partial V = \frac{Q}{A\epsilon_0} d \rightarrow Q = \frac{\epsilon_0 A}{d} V$$



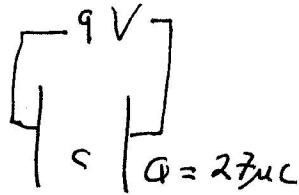
$$C = \frac{\sigma}{\epsilon_0}$$

$$\sigma = Q/A$$

$$\downarrow C_{11} = \frac{\epsilon_0 A}{d}$$

Capacitance is function of geometry and constants.

16.26

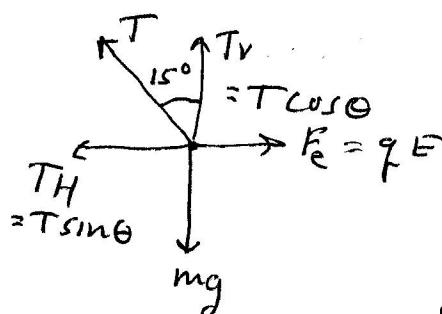
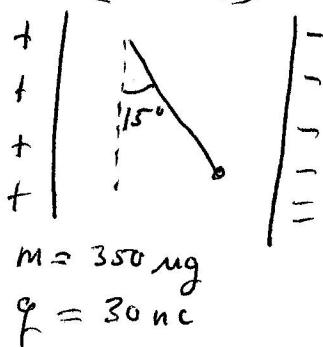


$$a) C = \frac{Q}{V} = \frac{27 \mu C}{9V} = 3.0 \mu F$$

16.7

$$b) Q = CV = (3.0 \mu F)(12V) = 36 \mu C$$

16.32 $d = 4 \text{ cm}$



$$\sum F_y = 0 = T_v - mg$$

$$\sum F_H = F_e - T_H$$

$$T_H = T \sin 15^\circ = qE$$

$$T_v = T \cos 15^\circ = mg$$

$$\therefore \tan 15^\circ = \frac{qE}{mg}$$

$$E = \frac{\Delta V}{d} \Rightarrow mg \tan 15^\circ = q \frac{\Delta V}{d}$$

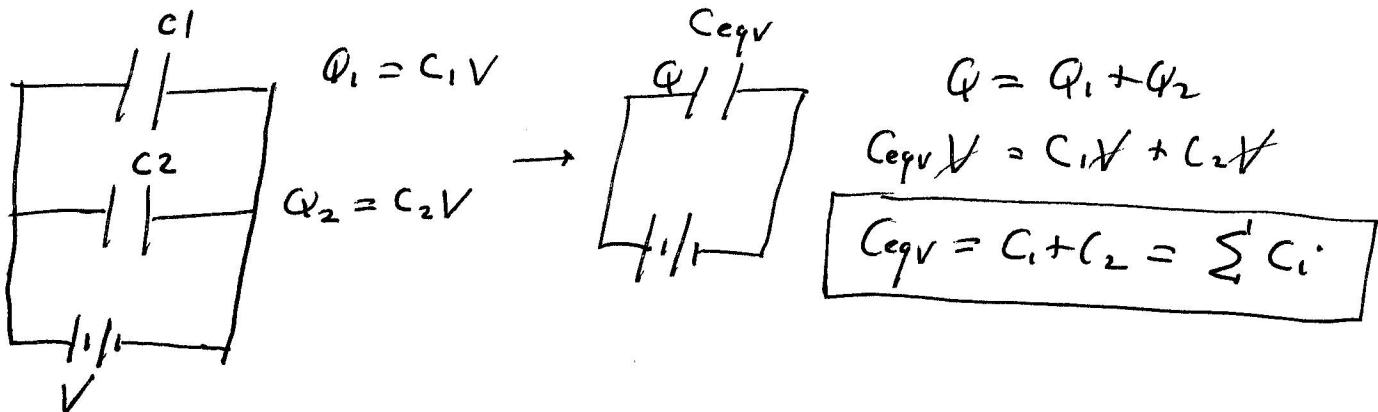
$$\Delta V = \frac{mg \tan 15^\circ d}{q} = \frac{(350 \times 10^{-6} \text{ kg} \times 10^3 \text{ N/C})(9.8) \tan 15^\circ (0.04 \text{ m})}{30 \times 10^{-9} \text{ C}}$$

$$\boxed{\Delta V = 1.23 \text{ V}}$$

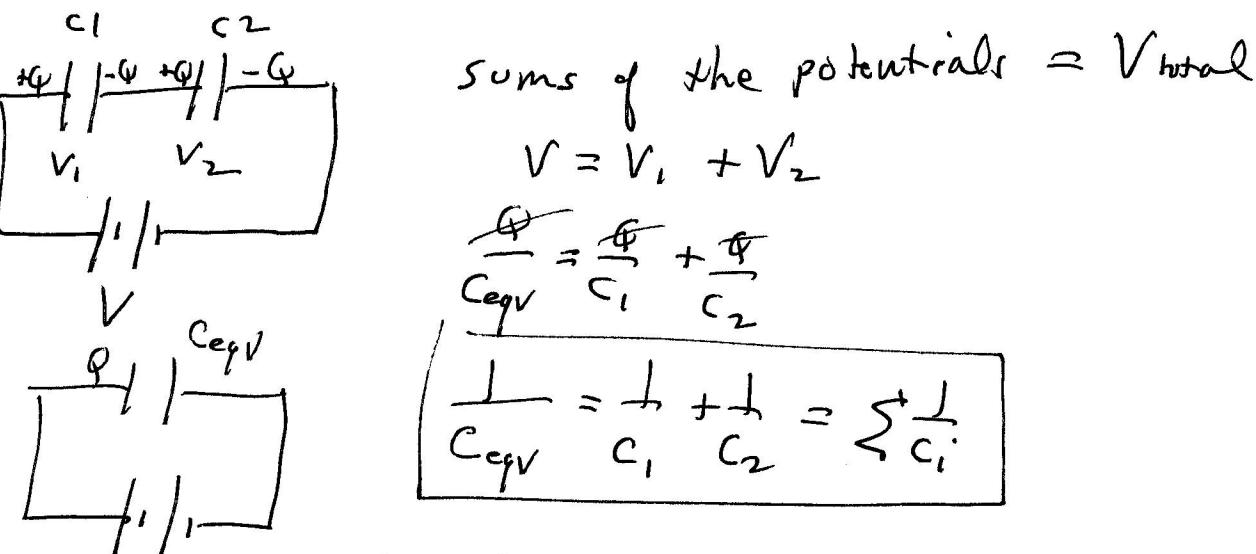
Series + Parallel Capacitors

$$Q = CV$$

Parallel \rightarrow capacitors have same potential difference



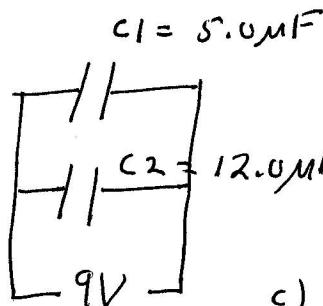
Series : Capacitors have the same charge



$$\frac{1}{C_{eqv}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2}{C_1 C_2} + \frac{C_1}{C_1 C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{eqv} = \frac{C_1 C_2}{C_1 + C_2}$$

16.34



$$a) C_{eqv} = C_1 + C_2 = 5 + 12 = 17 \mu F$$

b) in parallel, components have the same potential = $9V$

$$c) Q_1 = C_1 V = 5.0 \mu F \times 9V = 45.0 \mu C$$

$$Q_2 = C_2 V = (12 \mu F)(9V) = 108 \mu C$$

16.36 in parallel $C_1 + C_2 = 9.0 \mu F = C_{eqv}$

$$\text{in series } \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2 \mu F} = \frac{1}{C_{eqv}} \rightarrow C_2 = 9 \mu F - C_1$$

$$\frac{1}{C_1} + \frac{1}{(9 \mu F - C_1)} = \frac{1}{2 \mu F}$$

$$\times C_1 : 1 + \frac{C_1}{9 - C_1} = \frac{C_1}{2}$$

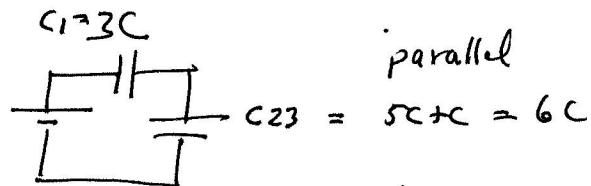
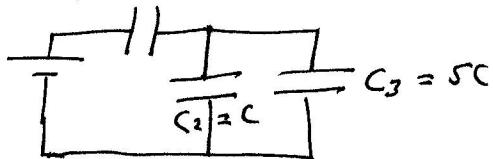
$$\times (9 - C_1) : 9 - C_1 + \cancel{C_1} = \underbrace{C_1 (9 - C_1)}_{2}$$

$$18 = 9C_1 - C_1^2 \rightarrow C_1^2 - 9C_1 + 18 = 0$$

$$(C_1 - 6)(C_1 - 3) = 0$$

$$\therefore C_1 = 6 \mu F \text{ or } 3 \mu F$$

$$C_2 = 3 \mu F \text{ or } 6 \mu F$$

16.40 $C_1 = 3C$  C_1 and C_{23} are in series

a) $C_{123} = \frac{C_1 \times C_{23}}{C_1 + C_{23}} = \frac{(3C)(6C)}{3C + 6C} = \frac{18C^2}{9C} = 2C$

b) for picture 2: $Q_1 = Q_{23}$ because they are in series

$$Q_{23} = Q_2 + Q_3$$

since the potential across C_2 = potential across C_3

$$(V_2 = V_3) = V_{23}$$

then $Q_3 = C_3 \cdot V_{23}$ and $Q_2 = C_2 \cdot V_{23}$

since $C_3 > C_2$ then $Q_3 > Q_2$

so $\boxed{Q_1 > Q_3 > Q_2}$

c) $V_1 = \frac{Q_1}{3C}$, $V_{23} = \frac{Q_{23}}{6C} = \frac{Q_1}{6C}$ $\therefore V_1 > V_{23}$

$\therefore \boxed{V_1 > V_2 = V_3}$

d) if $C_3 \uparrow$ (increases) then $C_{23} \uparrow$ [increase C_3 by 2
 $\frac{3+8}{3+8} = \frac{24}{11} > 2$]
 then $C_{123} \uparrow$

since $C_{123} \uparrow$, $Q_{tot} = C_{123} \times V$ must increase

$$\text{also } Q_1 = Q_{23} \uparrow$$

$$V_1 + V_{23} = \text{const}$$

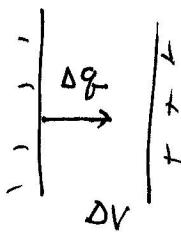
so since $Q_1 \uparrow \Rightarrow V_1 = \frac{Q_1}{C_1} \uparrow \rightarrow$ if $V_1 \uparrow$ then $V_{23} \downarrow$

$$Q_2 = C_2 V_{23}$$

$$Q_3 = C_3 V_{23} \quad \text{if} \quad C_3 \uparrow \text{ then } Q_3 \uparrow$$

and so Q_2 must \downarrow

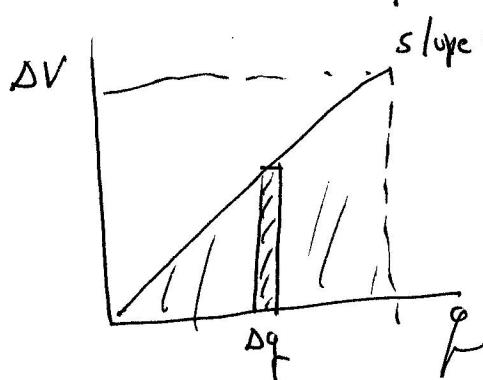
charging up a capacitor



the work needed to move a charge Δq across the plate

$$\Delta W = \Delta q \cdot \Delta V$$

the more charge that is moved, the greater the potential difference becomes



$\Delta q \cdot \Delta V = \text{area of rectangle}$,
so the total work to
move q : $0 \rightarrow q$
through ΔV : $0 \rightarrow \Delta V$
= area under triangle

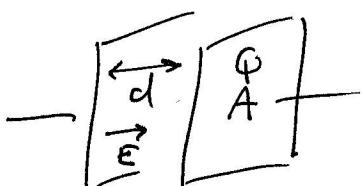
$$\text{Area} = \frac{1}{2} \text{base} \times \text{height}$$

$$\text{work} = \Delta U + 0 \quad U = \boxed{\Delta U = \frac{1}{2} q \Delta V}$$

$$\begin{aligned}\Delta U &= \frac{1}{2} q \Delta V \\ &\Rightarrow \frac{1}{2} C (\Delta V)^2 \\ &\approx \frac{1}{2} \frac{q^2}{C}\end{aligned}$$

$$\begin{aligned}q &= CV \\ \Delta V &= \frac{q}{C}\end{aligned}$$

$$\text{Energy density} = \frac{U}{\text{vol}}$$



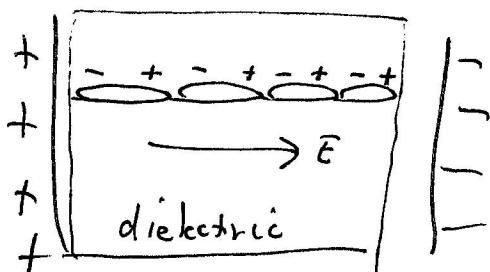
$$\Delta V = Ed$$

$$\frac{U}{\text{vol}} = \frac{\frac{1}{2} CV^2}{A \cdot d} = \frac{\frac{1}{2} \left(\frac{\epsilon_0 A}{d}\right)(Ed)^2}{Ad} = \frac{\frac{1}{2} \frac{\epsilon_0 A}{d} E^2 d^2}{Ad}$$

$$\boxed{\frac{U}{\text{vol}} = \frac{1}{2} \epsilon_0 E^2}$$

Dielectrics (insulators)

If capacitors have dielectrics between the conductors, the electric field is altered and the capacitance increases.



In the mechanical model, the dielectric is made of polarized molecules that line up in the electric field. The molecules at the edges

"neutralize" the surface charge density on the capacitor plates, reducing the effective surface charge density, σ .

$$E = \frac{\sigma}{\epsilon_0} \quad \text{so if } \sigma \text{ decreases, the } E \text{ decreases}$$

$$\text{and } V = E d, \text{ so potential decreases,}$$

meaning more charge can be placed on the plates
and capacitance increases.

We say the permittivity is changed

$$\epsilon_0 \rightarrow \epsilon_k = k \epsilon_0$$

$$C_0 = \frac{\epsilon_0 A}{d} \rightarrow C_k = \frac{\epsilon_k A}{d} = k \frac{\epsilon_0 A}{d} = k C_0$$

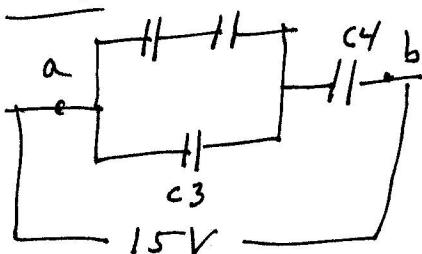
k = dielectric constant

= 1.000000 for vacuum

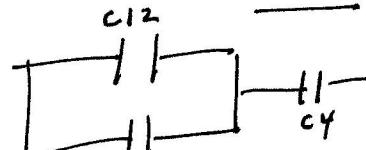
= 1.000 84 for air

= 3.3 for paper

16.44



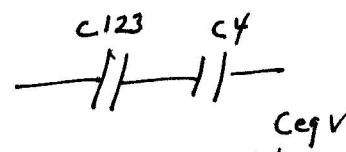
$$\begin{aligned}C_1 &= 15\mu F \\C_2 &= 3\mu F \\C_3 &= 6\mu F \\C_4 &= 20\mu F\end{aligned}$$

a) combine C_1 and C_2 in series

$$C_{12} = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{15 \times 3}{15 + 3} = \frac{45}{18} = 2.5\mu F$$

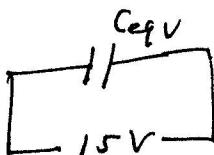
b) C_{12} and C_3 are in parallel

$$C_{123} = C_{12} + C_3 = 2.5\mu F + 6\mu F = 8.5\mu F$$

c) C_{123} and C_4 are in series

$$C_{eqv} = \frac{C_{123} \times C_4}{C_{123} + C_4} = \frac{8.5 \times 20}{8.5 + 20} = \frac{170}{28.5} = 5.96\mu F$$

$$Q_{eqv} = C_{eqv} \times V$$



$$Q_{eqv} = 5.96\mu F \times 15V = 89.5\mu C$$

i) since C_{123} and C_4 are in series they have the same charge

$$Q_4 = Q_{123} = Q_{eqv} \Rightarrow Q_4 = 89.5\mu C$$

$$\text{i)} V_4 = \frac{Q_4}{C_4} = \frac{89.5\mu C}{20\mu F} = 4.47V \therefore V_{123} = 15V - 4.47V = 10.5V$$

ii) since C_{123} is from two caps (C_1 and C_2) in parallel, they have the same potential difference

$$V_{12} = V_3 = V_{123} = 10.5V$$

$$\text{iv)} Q_3 = C_3 V_3 = 6\mu F \times 10.5V = 63.2\mu C = Q_3$$

$$\text{v)} Q_{123} = 89.5\mu C = Q_{12} + Q_3 \Rightarrow Q_{12} = 89.5\mu C - 63.2\mu C$$

$$Q_{12} = 26.3\mu C$$

vi) Q_{12} is from two caps (C_1 and C_2) in series, so they have the same charge

$$Q_1 = Q_2 = Q_{12} = 26.3\mu C$$

16.50

a)

$$E_{\max} = 3.0 \times 10^6 \frac{V}{m}$$

$$A = 5.00 \text{ cm}^2 \quad C_n = \frac{\epsilon_0 A}{d} = K \frac{\epsilon_0 A}{d}$$

$$Q_{\max} = C_n V_{\max} = \frac{K_{\text{air}} \epsilon_0 A}{d} (E_{\max} d)$$

for air $K_{\text{air}} = 1.000$

$$Q_{\max} = \frac{(1.000)(8.85 \times 10^{-12})(5 \times 10^{-4} \text{ m}^2)(3 \times 10^6)}{(1.000)(8.85 \times 10^{-12})} (3 \times 10^6)$$

$$Q_{\max} = 1.33 \times 10^{-8} \text{ C}$$

b) with polystyrene, $E_{\max} = 24.0 \times 10^6 \text{ V/m}$, $K_p = 2.56$

$$Q_{\max} = (2.56)(8.85 \times 10^{-12})(5 \times 10^{-4} \text{ m}^2)(24 \times 10^6 \text{ V/m}) \\ = 2.72 \times 10^{-7} \text{ C}$$